# 1.1 GETTING STARTED

### Objectives

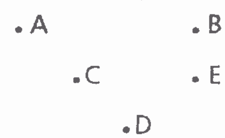
*After studying this section, you will be able to*

- Recognize points
- Recognize lines
- Recognize line segments
- Recognize rays
- Recognize angles
- Recognize triangles

## *Part One: Introduction*

### Points

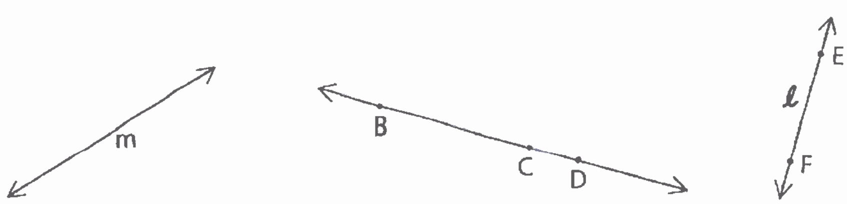
In the diagram at the right, five ***points*** are represented by five dots. The names of the points are A, B, C, D, and E. (We use capital letters to name points.)




### Lines

The diagram below represents three ***lines***. Lines are made up of points and are straight. The arrows on the ends of the figures show that the lines extend infinitely far in both directions.

*All lines are straight and extend infinitely far in both directions.*




- The line on the left is called line m.
- Since we can name a line in terms of any two points on it, the line in the middle can be called by a variety of names.

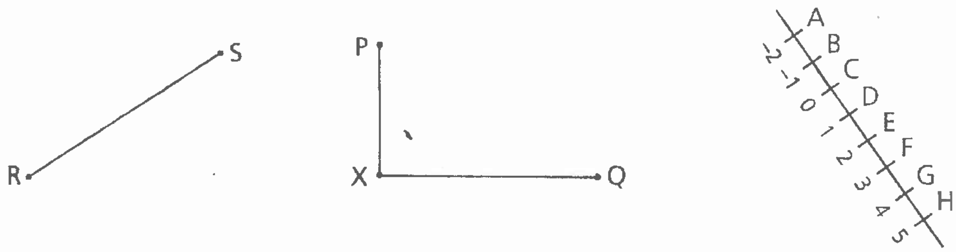
$\overleftrightarrow{BD}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{DC}$

- The line on the right can be called by any of three names.

line $\ell$ $\overleftrightarrow{EF}$ $\overleftrightarrow{FE}$

## Line Segments

The following diagram represents several ***line segments***, or simply ***segments.*** Like lines, segments are made up of points and are straight. A segment, however, has a definite beginning and end.

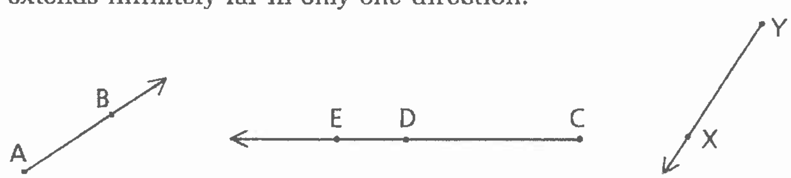



A segment is named in terms of its two ***endpoints***

- The segment on the left can be called either $\overline{RS}$ or $\overline{SR}$.
- In the middle figure there are two segments. The vertical (up-and-down) segment can be called either $\overline{PX}$ or $\overline{XP}$. The horizontal (crosswise) segment can also be named in two ways. Can you name these two ways?
- How might we name the segment whose endpoints have coordinates 3 and 0 in the figure on the right?

## Rays

In the diagram below, three ***rays*** are represented. Rays, like lines and segments, are made up of points and are straight. A ray differs from a line or a segment in that it begins at an endpoint and then extends infinitely far in *only* one direction.



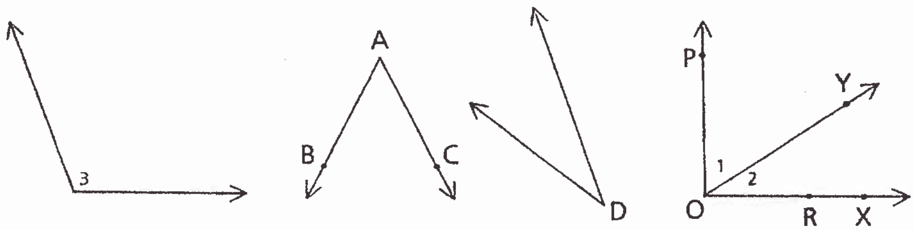

When we name a ray, we must name the endpoint first so that it is clear where the ray begins.

- The ray on the left is called $\overrightarrow{AB}$.
- The ray in the middle can be called $\overrightarrow{CD}$ or $\overrightarrow{CE}$. (As long as the endpoint is given first, any other point on the ray can be used in its name.)
- The ray on the right can be named in only one way. Do you know what its name is?

## Angles

Two rays that have the same endpoint form an ***angle***.

**Definition** An ***angle*** is made up of two rays with a common endpoint. This point is called the ***vertex*** of the angle. The rays are called ***sides*** of the angle.




- In the diagram above, the angle on the left is called ∠3. The 3 placed inside the angle near the vertex names it.
- The second angle in the diagram can be called by any of three names.

  ∠BAC ∠CAB ∠A

  (Notice that when we use three letters, the vertex must be named in the middle.)
- The third angle is called ∠D.
- In the last figure above, there are three angles. Can you tell which angle is ∠O? Because names might refer to more than one angle in a diagram, we *never name an angle in a way that could result in confusion*.

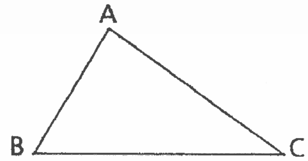
  ∠1 can also be called ∠POY or ∠YOP.

  ∠2 can also be called ∠YOR, ∠YOX, ∠ROY, or ∠XOY.

  The other angle in this figure can be named ∠POR. See if you can find three other names for this angle.

## Triangles

We shall call the following figure ***triangle*** ABC (△ABC).




A triangle has three segments as its sides. You may wonder whether we can talk about an ∠B in the triangle, since there are no arrows in the diagram. The answer is yes. We shall often talk about rays, lines, and angles in a diagram of a triangle. So a triangle not only has three sides but has three angles as well. Can you name the angle at the top of the triangle shown on the preceding page in three ways?

The triangle is the ***union*** (∪) of three segments.

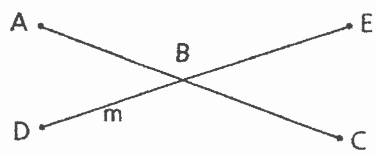
$$\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{AC}$$

The ***intersection*** (∩) of any two sides is a ***vertex*** of the triangle.

$$\overline{AB} \cap \overline{BC} = B$$

# *Part Two: Sample Problems*

**Problem 1**




**a** *How many lines are shown? (Imagine that there are arrows in the diagram.)*

**b** *Name these lines.*

**c** *Where do* $\overleftrightarrow{AC}$ *and* $\overleftrightarrow{DE}$ *intersect?*

**d** *Where does* $\overrightarrow{AC}$ *intersect* $\overleftrightarrow{BC}$*?* ($\overrightarrow{AC} \cap \overleftrightarrow{BC}$ = __?__)

**e** *What is the union of* $\overrightarrow{BA}$ *and* $\overrightarrow{BD}$*?* ($\overrightarrow{BA} \cup \overrightarrow{BD}$ = __?__)

**Answers**

**a** 2

**b** Line m, $\overleftrightarrow{DB}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{BE}$, $\overleftrightarrow{EB}$, or $\overleftrightarrow{ED}$; $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, or $\overleftrightarrow{CB}$

**c** B

**d** $\overrightarrow{AC}$ (Remember sets? If P and Q are two sets of points, then P ∩ Q = {all points in P and in Q}.)

**e** ∠ABD (P ∪ Q = {all points in P or in Q or in both}.)

# 1.3 COLLINEARITY, BETWEENNESS, AND ASSUMPTIONS

## Objectives

*After studying this section, you will be able to*

- Recognize collinear and noncollinear points
- Recognize when a point can be said to be between two others
- Recognize that each side of a triangle is shorter than the sum of the other two sides
- Correctly interpret geometric diagrams

## *Part One: Introduction*

### Collinearity

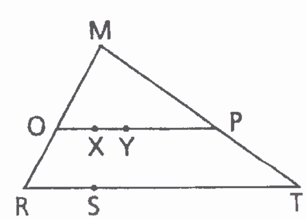
It is often useful to know that a group of points lie on the same line.

**Definition** Points that lie on the same line are called ***collinear***. Points that do not lie on the same line are called ***noncollinear***.


COLLINEAR POINTS


NON-COLLINEAR POINTS

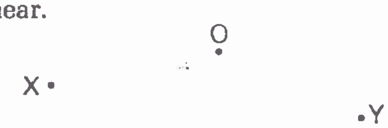
In the diagram at the right, R, S, and T are collinear points. P, O, and X are also collinear. M, O, X, and Y are noncollinear.




### Betweenness of Points

In order for us to say that a point is between two other points, all three of the points must be collinear.


T is between A and R




We do *not* say O is between X and Y

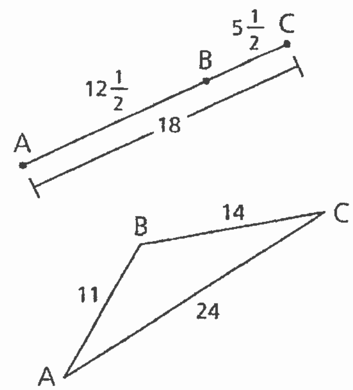
## Triangle Inequality

For any three points, there are only two possibilities:

1 They are collinear. (One point is between the other two. Two of the distances add up to the third.)

2 They are noncollinear. (The three points determine a triangle.)

Notice that in the triangle, $14 + 11 > 24$. This is an example of an important characteristic of triangles: *The sum of the lengths of any two sides of a triangle is always greater than the length of the third.*



## Assumptions from Diagrams

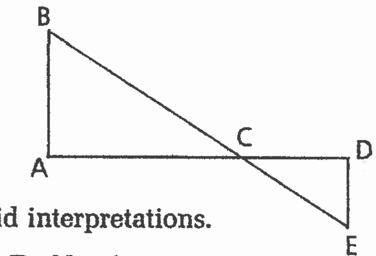
You may wonder what you should and should not assume when you look at a diagram. The chart below gives the general rules you should follow as you work with this book. (There are, however, occasional exceptions, as in Section 1.2, problem 19.)

**How to Interpret a Diagram**

| You Should Assume | You Should Not Assume |
|---|---|
| Straight lines and angles | Right angles |
| Collinearity of points | Congruent segments |
| Betweenness of points | Congruent angles |
| Relative positions of points | Relative sizes of segments and angles |

The following example will help you understand what assumptions can be made.

**Example** *Given: Diagram as shown*
*Question: What should we assume?*

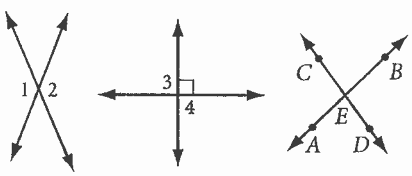



The following are some of the many valid interpretations.

| Do Assume | Do Not Assume |
|---|---|
| $\overleftrightarrow{ACD}$ and $\overleftrightarrow{BCE}$ are straight lines. | $\angle BAC$ is a right $\angle$. |
| $\angle BCE$ is a straight angle. | $\overline{CD} \cong \overline{DE}$ |
| C, D, and E are noncollinear. | $\angle B \cong \angle E$ |
| C is between B and E. | $\angle CDE$ is an obtuse angle. |
| E is to the right of A. | $\overline{BC}$ is longer than $\overline{CE}$. |

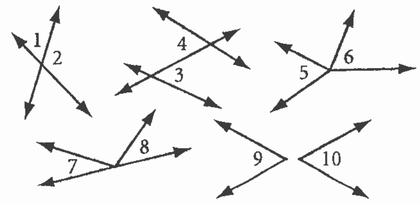
Reread and study the chart and the example carefully, for it is important that you know what to assume from a diagram.

## Pair of Vertical Angles



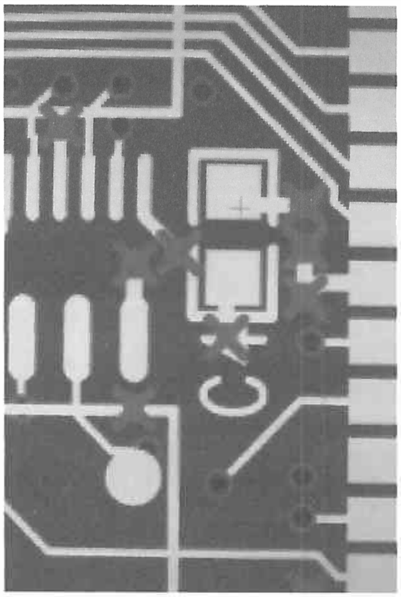

Pairs of vertical angles:

∠1 and ∠2
∠3 and ∠4
∠AED and ∠BEC
∠AEC and ∠DEB



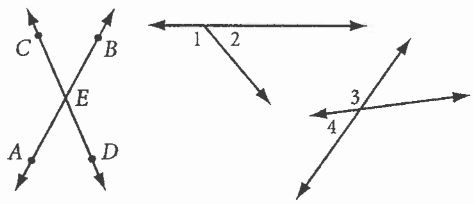

Not pairs of vertical angles:

∠1 and ∠2
∠3 and ∠4
∠5 and∠6
∠7 and ∠8
∠9 and ∠10



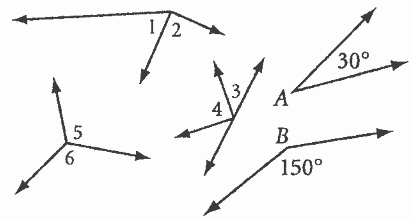
What types of angles or angle pairs do you see in this magnified view of a computer chip?

## Linear Pair of Angles




Linear pairs of angles:

∠1 and ∠2
∠3 and ∠4
∠AED and ∠AEC
∠BED and ∠DEA

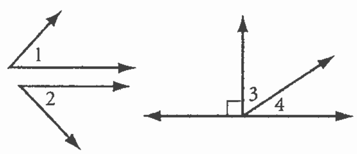



Not linear pairs of angles:

∠1 and ∠2
∠3 and ∠4
∠5 and ∠6
∠A and ∠B

## Pair of Complementary Angles

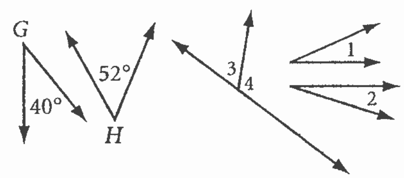
$m\angle 1 + m\angle 2 = 90°$




Pairs of complementary angles:

∠1 and ∠2
∠3 and ∠4

$m\angle 1 + m\angle 2 \neq 90°$

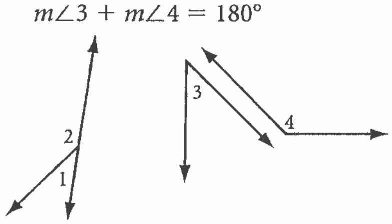



Not pairs of complementary angles:

∠G and ∠H ∠1 and ∠2
∠3 and ∠4

## Pair of Supplementary Angles

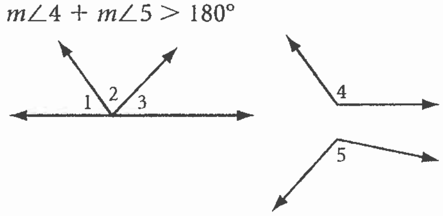
$m\angle 3 + m\angle 4 = 180°$




Pairs of supplementary angles:

∠1 and ∠2
∠3 and ∠4

$m\angle 4 + m\angle 5 > 180°$




Not pairs of supplementary angles:

∠1, ∠2, and ∠3
∠4 and ∠5

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# 4.5 INTRODUCTION TO PARALLEL LINES

## Objectives

*After studying this section, you will be able to*

- Recognize planes
- Recognize transversals
- Identify the pairs of angles formed by a transversal
- Recognize parallel lines

## *Part One: Introduction*

### Planes

In order to explain parallel lines adequately, we must first acquaint you with the meaning of ***plane***.

**Definition** A ***plane*** is a surface such that if any two points on the surface are connected by a line, all points of the line are also on the surface.

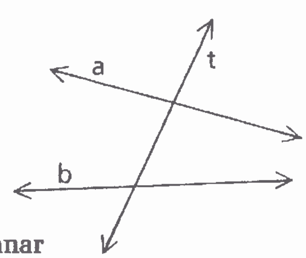
A plane has only two dimensions—length and width. Both the length and the width are infinite. A plane has no thickness.

**Definition** If points, lines, segments, and so forth, lie in the same plane, we call them ***coplanar***. Points, lines, segments, and so forth, that do not lie in the same plane are called ***noncoplanar***.

Planes are discussed more fully in Chapter 6.

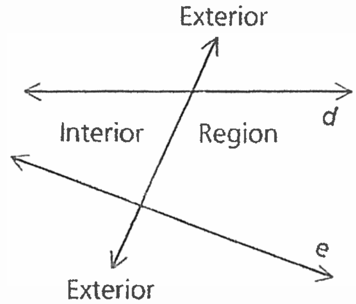
### Transversals

In the figure, line t is a ***transversal*** of lines a and b.

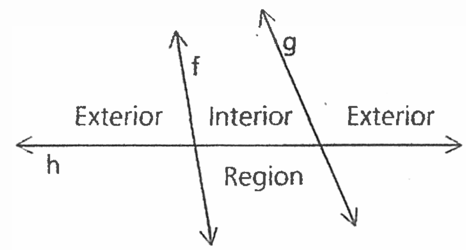



**Definition** A ***transversal*** is a line that intersects two coplanar lines in two distinct points.

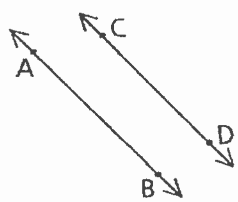
The region between lines d and e is the ***interior*** of the figure. The rest of the plane is the ***exterior***.

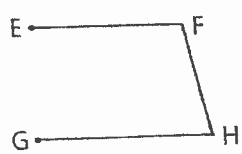


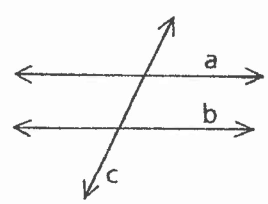

The diagram of lines f and g cut by transversal h provides another illustration of the regions formed by two lines and a transversal.




## Parallel Lines




Above are three illustrations of ***parallel*** (∥) lines. We write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$, and a ∥ b.

**Definition** ***Parallel lines*** are two coplanar lines that do not intersect.

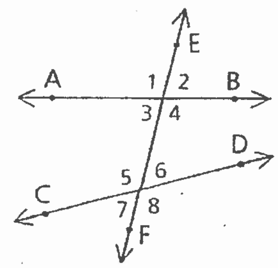
We shall also call segments or rays parallel if they are parts of parallel lines. For example, we can say that in the preceding diagrams $\overline{AB} \parallel \overline{CD}$ and $\overline{EF} \parallel \overline{GH}$.

There are many lines that do not intersect yet are not parallel. *To be parallel, lines must be coplanar.* In Chapter 6, lines that are noncoplanar and nonintersecting are defined as *skew* lines.

## Angle Pairs Formed by Transversals

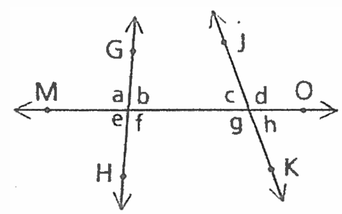
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are cut by transversal $\overleftrightarrow{EF}$.

The two pairs of ***alternate interior angles*** are 3 and 6, 4 and 5.
The two pairs of ***alternate exterior angles*** are 1 and 8, 2 and 7.
The four pairs of ***corresponding angles*** are 1 and 5, 2 and 6, 3 and 7, 4 and 8.

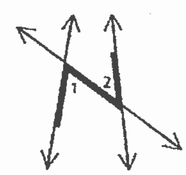



$\overleftrightarrow{GH}$ and $\overleftrightarrow{JK}$ are cut by transversal $\overleftrightarrow{MO}$.

The alternate interior angles are b and g, f and c.
The alternate exterior angles are a and h, e and d.
The corresponding angles are a and c, b and d, e and g, f and h.

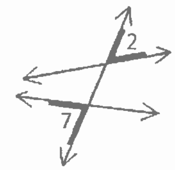



**Definition** ***Alternate interior angles*** are a pair of angles formed by two lines and a transversal. The angles must both lie in the interior of the figure, must lie on alternate sides of the transversal, and must have different vertices.

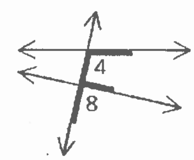



Look for an N or Z shape.

**Definition** *Alternate exterior angles* are a pair of angles formed by two lines and a transversal. The angles must both lie in the exterior of the figure, must lie on alternate sides of the transversal, and must have different vertices.

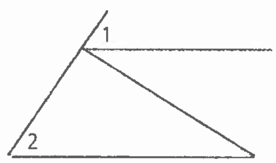



**Definition** *Corresponding angles* are a pair of angles formed by two lines and a transversal. One angle must lie in the interior of the figure, and the other must lie in the exterior. The angles must lie on the same side of the transversal but have different vertices.

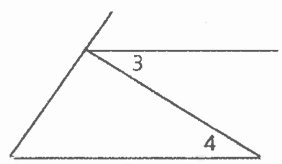


Look for an F shape.

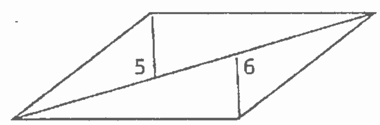
It is important to be able to recognize these pairs of angles when they appear in figures made up of a number of segments. In each of the following examples, the segment corresponding to the transversal is shown in red, and the segments corresponding to the lines it cuts are shown in blue.



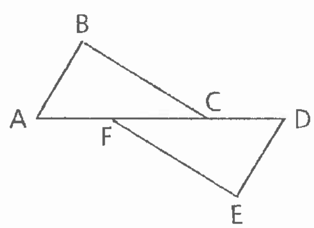
$\angle 1$ and $\angle 2$ are corresponding $\angle$s.



$\angle 3$ and $\angle 4$ are alternate interior $\angle$s.

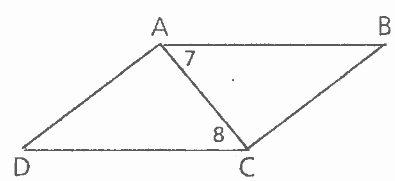


$\angle 5$ and $\angle 6$ are alternate exterior $\angle$s.



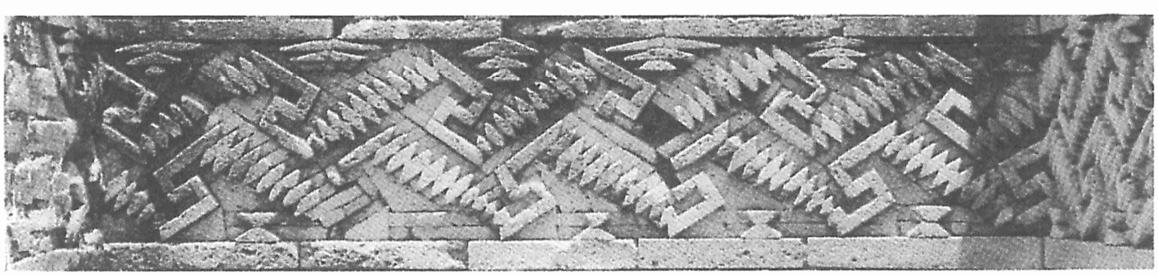
$\angle$BCA and $\angle$DFE are alternate interior $\angle$s.

$\angle$BCD and $\angle$EFA are alternate exterior $\angle$s.



$\angle 7$ and $\angle 8$ are alternate interior $\angle$s.

Can you find a pair of alternate interior $\angle$s formed by $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ with transversal $\overleftrightarrow{AC}$?



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# 6.1 RELATING LINES TO PLANES

## Objectives

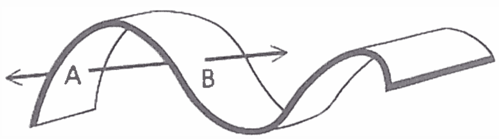
*After studying this chapter, you will be able to*

- Understand basic concepts relating to planes
- Identify four methods of determining a plane
- Apply two postulates concerning lines and planes

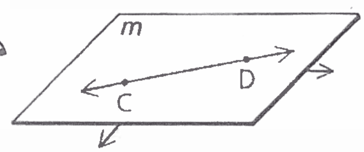
## *Part One: Introduction*

### Preliminary Concepts

Recall the definition of *plane* from Section 4.5: *A plane is a surface such that if any two points on the surface are connected by a line, all points of the line are also on the surface.* Because a surface has no thickness, a plane must be "flat" if it is to contain the straight lines determined by all pairs of points on it. It must also be infinitely long and wide. Thus, a plane has only two dimensions, length and width.




A surface that is not a plane

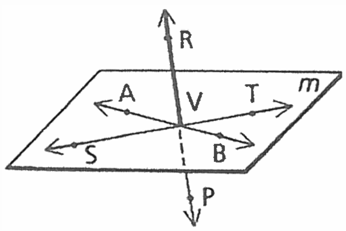



Plane surface

A plane is frequently drawn as shown in the right-hand diagram above. In this case, the diagram represents part of a horizontal plane, with the edges nearest to you darkened. A plane can be named by placing a single lowercase letter in one of the corners.

It is important to understand that although our picture of a plane has edges and corners, an actual plane has neither and should be thought of as infinite in length and width.

You may recall the following definitions from Section 4.5: *If points, lines, segments, and so forth, lie in the same plane, we call them coplanar. Points, lines, segments, and so forth, that do not lie in the same plane are called noncoplanar.* In the diagram on the next page, $\overleftrightarrow{AB}$ and $\overleftrightarrow{ST}$ lie in plane m. $\overleftrightarrow{RP}$ does not lie in the plane but intersects m at V.




A, B, S, T, and V are *coplanar* points.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{ST}$ are *coplanar* lines.
$\overline{AB}$ and $\overline{ST}$ are *coplanar* segments.

A, B, S, T, and R are *noncoplanar* points.
$\overleftrightarrow{AB}$, $\overleftrightarrow{ST}$, and $\overleftrightarrow{RP}$ are *noncoplanar* lines.
$\overline{AB}$, $\overline{ST}$, and $\overline{RP}$ are *noncoplanar* segments.

**Definition** The point of intersection of a line and a plane is called the ***foot*** of the line.

In the preceding diagram, V is the foot of $\overleftrightarrow{RP}$ in plane m.

# *Part One: Introduction*

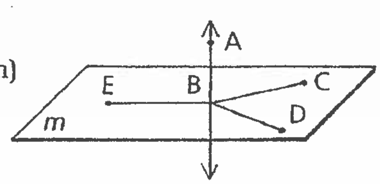
## A Line Perpendicular to a Plane

What does it mean to say that a line is perpendicular to a plane? Think about that for a moment and then read the following formal definition.

**Definition** A line is perpendicular to a plane if it is perpendicular to every one of the lines in the plane that pass through its foot.

Observe that we now have two kinds of perpendicularity:

1 Between two lines ($\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$)
2 Between a line and a plane ($\overleftrightarrow{AB} \perp$ m)




The definition above is a very powerful statement because of the words *every one*. If we are given that $\overleftrightarrow{AB} \perp$ m (in the diagram above), we can draw *three* conclusions:

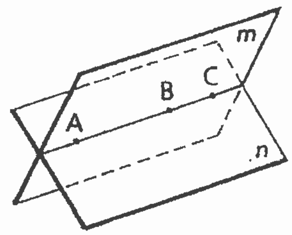
$\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$ $\overleftrightarrow{AB} \perp \overleftrightarrow{BE}$

## Four Ways to Determine a Plane

In Chapter 3, you learned that two points determine a line. We would now like to find conditions under which a plane is determined.

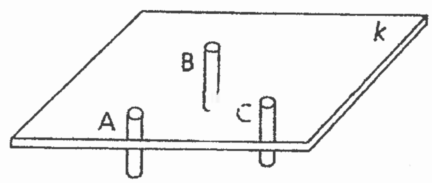
One point obviously does not determine a plane, since infinitely many planes pass through a single point.

The diagram at the right shows that two points also do not determine a unique plane. It shows two different planes, m and n, each of which contains both point A and point B. The same diagram shows that three points—A, B, and C—do not determine a plane if the three points are collinear.




If the three points are noncollinear, however, they do determine a plane.

There is one and only one plane that contains the three noncollinear points A, B, and C. This plane can be called either plane ABC or plane k.

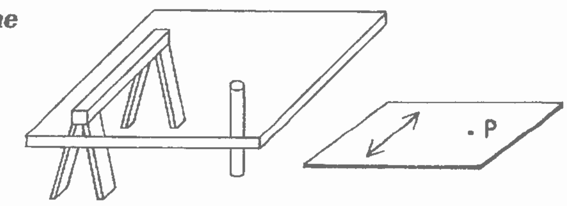



The preceding observations suggest an important postulate.

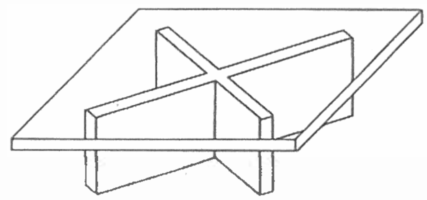
**Postulate** ***Three noncollinear points determine a plane.***

There are other ways of determining a plane. The following three are stated as theorems.

**Theorem 45** ***A line and a point not on the line determine a plane.***

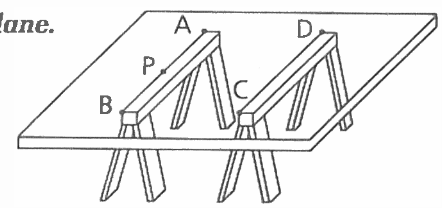



**Theorem 46** ***Two intersecting lines determine a plane.***



The proofs of Theorems 45 and 46 are asked for in Problem Set B.

**Theorem 47** ***Two parallel lines determine a plane.***

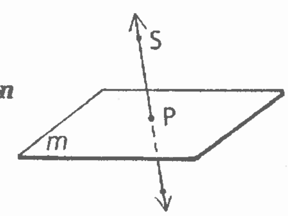



*Proof:* If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, then according to the definition of *parallel lines,* they lie in a plane. We need to show that they lie in *only one* plane. If P is any point on $\overleftrightarrow{AB}$, then according to Theorem 45, there is only one plane containing P and $\overleftrightarrow{CD}$. Thus, there is only one plane that contains $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, because every plane containing $\overleftrightarrow{AB}$ contains P.

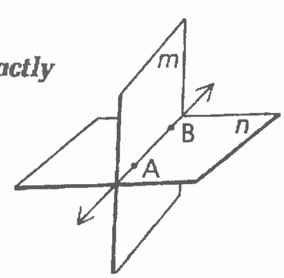
## Two Postulates Concerning Lines and Planes

We shall assume the following two statements.

**Postulate** ***If a line intersects a plane not containing it, then the intersection is exactly one point.***




**Postulate** ***If two planes intersect, their intersection is exactly one line.***



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# 6.3 BASIC FACTS ABOUT PARALLEL PLANES

## Objectives

*After studying this section, you will be able to*

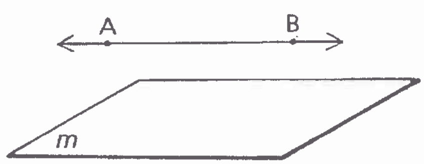
- Recognize lines parallel to planes, parallel planes, and skew lines
- Use properties relating parallel lines and planes

## *Part One: Introduction*

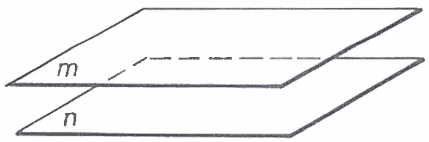
### Lines Parallel to Planes, Parallel Planes, Skew Lines

Since we examined the concept of parallel lines in Chapter 4, it seems logical now to investigate the possibilities of a line being parallel to a plane and of two planes being parallel to each other.

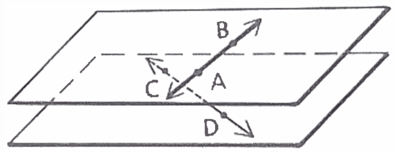
**Definition** A line and a plane are parallel if they do not intersect.




**Definition** Two planes are parallel if they do not intersect.




The diagram at the right shows two lines located in two parallel planes. Although the planes are parallel, the lines are not parallel, because A, B, C, and D do not determine a plane. Such lines are said to be ***skew***.



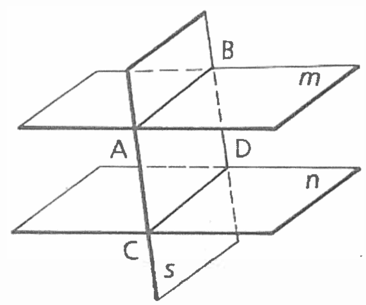
**Definition** Two lines are ***skew*** if they are not coplanar.

You will see that parallelism in space is very similar to parallelism in a plane. There are, however, a few notable differences. For example, there are no skew planes. Planes are either intersecting or parallel.

The following theorem is basic to the understanding of parallelism in space.

**Theorem 49** ***If a plane intersects two parallel planes, the lines of intersection are parallel.***

Given: $m \parallel n$;
s intersects
m and n in lines
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$




*Proof:* We know that $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are coplanar, since they both lie in plane s. Also, they cannot intersect each other, because one lies in plane m and the other lies in plane n—two planes that, being parallel, have no intersection. Thus, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ by the definition of *parallel lines.*

## Properties Relating Parallel Lines and Planes

There are numerous properties relating lines and planes in space, many of which are similar to the theorems about parallel lines you have already seen. We will present some of these properties without their proofs.

### Parallelism of Lines and Planes

1 If two planes are perpendicular to the same line, they are parallel to each other.
2 If a line is perpendicular to one of two parallel planes, it is perpendicular to the other plane as well.
3 If two planes are parallel to the same plane, they are parallel to each other.
4 If two lines are perpendicular to the same plane, they are parallel to each other.
5 If a plane is perpendicular to one of two parallel lines, it is perpendicular to the other line as well.