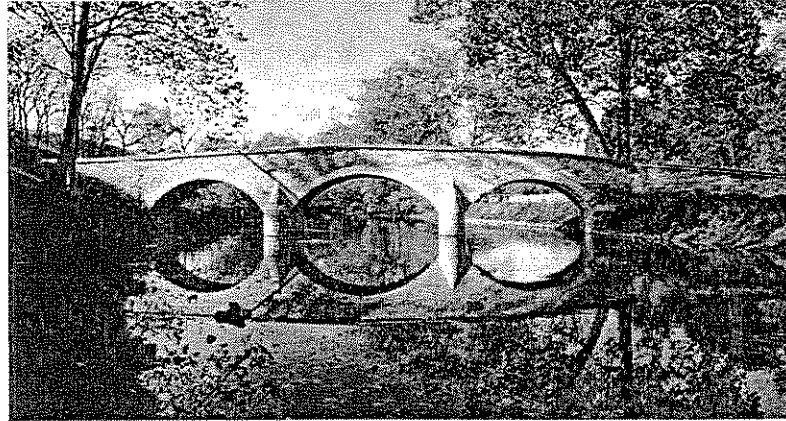


# 3.2

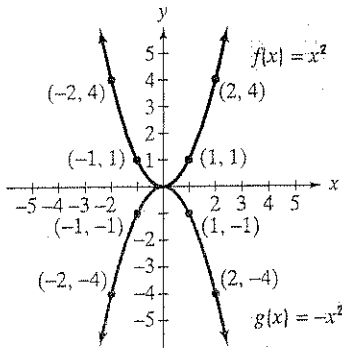
## Reflections of Graphs



This photograph shows a reflection of an old bridge in a Maryland river. This perfect reflection occurs because the surface of the water is absolutely still. A mild breeze rippling the water's surface would distort the reflection.

Is it possible for graphs to have mirror-like qualities? Yes. **Figure 1.57** shows the graphs of  $f(x) = x^2$  and  $g(x) = -x^2$ . The graph of  $g$  is a **reflection about the  $x$ -axis** of the graph of  $f$ . For corresponding values of  $x$ , the  $y$ -coordinates of  $g$  are the opposites of the  $y$ -coordinates of  $f$ . In general, the graph of  $y = -f(x)$  reflects the graph of  $f$  about the  $x$ -axis. Thus, the graph of  $g$  is a reflection of the graph of  $f$  about the  $x$ -axis because

$$g(x) = -x^2 = -f(x).$$



**Figure 1.57** Reflection about the  $x$ -axis

### Reflection about the $x$ -Axis

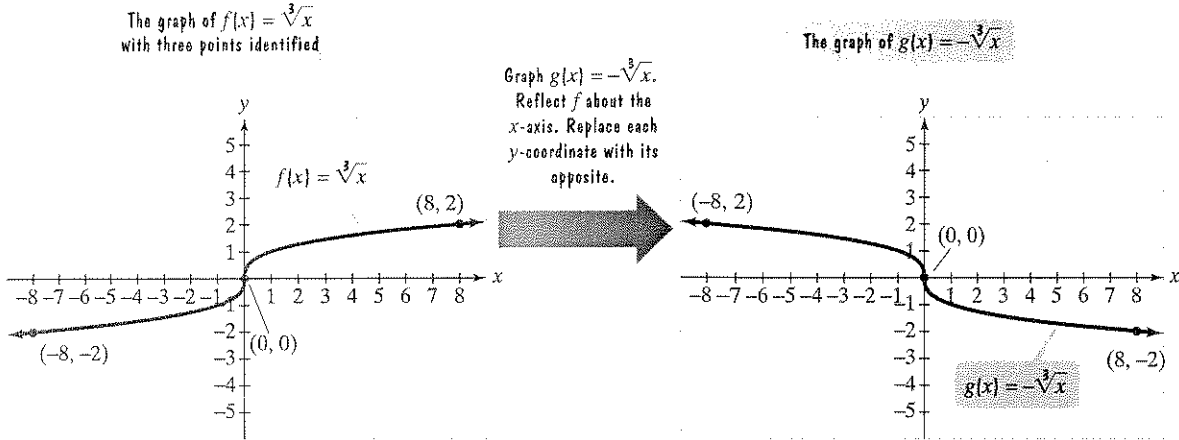
The graph of  $y = -f(x)$  is the graph of  $y = f(x)$  reflected about the  $x$ -axis.

**EXAMPLE 4** Reflection about the x-Axis

Use the graph of  $f(x) = \sqrt[3]{x}$  to obtain the graph of  $g(x) = -\sqrt[3]{x}$ .

**Solution** Compare the equations for  $f(x) = \sqrt[3]{x}$  and  $g(x) = -\sqrt[3]{x}$ . The graph of  $g$  is a reflection about the  $x$ -axis of the graph of  $f$  because

$$g(x) = -\sqrt[3]{x} = -f(x).$$



**Check Point 4** Use the graph of  $f(x) = |x|$  to obtain the graph of  $g(x) = -|x|$ .

It is also possible to reflect graphs about the  $y$ -axis.

**Reflection about the y-Axis**

The graph of  $y = f(-x)$  is the graph of  $y = f(x)$  reflected about the  $y$ -axis.

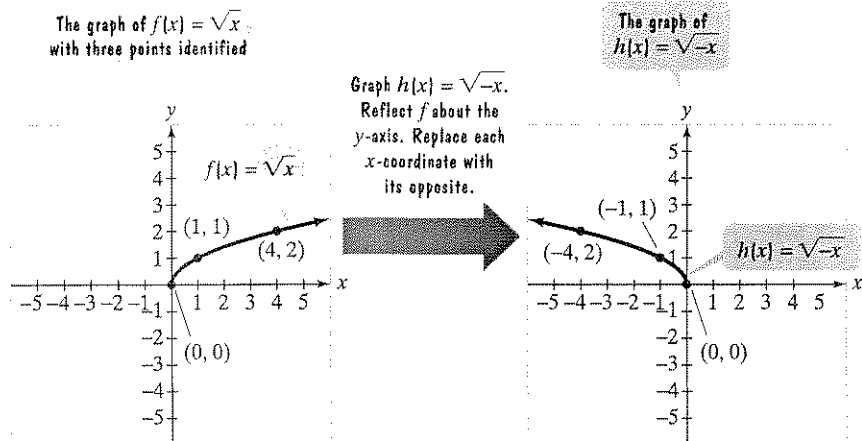
For each point  $(x, y)$  on the graph of  $y = f(x)$ , the point  $(-x, y)$  is on the graph of  $y = f(-x)$ .

**EXAMPLE 5** Reflection about the y-Axis

Use the graph of  $f(x) = \sqrt{x}$  to obtain the graph of  $h(x) = \sqrt{-x}$ .

**Solution** Compare the equations for  $f(x) = \sqrt{x}$  and  $h(x) = \sqrt{-x}$ . The graph of  $h$  is a reflection about the  $y$ -axis of the graph of  $f$  because

$$h(x) = \sqrt{-x} = f(-x).$$



**Check Point 5** Use the graph of  $f(x) = \sqrt[3]{x}$  to obtain the graph of  $h(x) = \sqrt[3]{-x}$ .