3.3

5 Use vertical stretching and shrinking to graph functions.

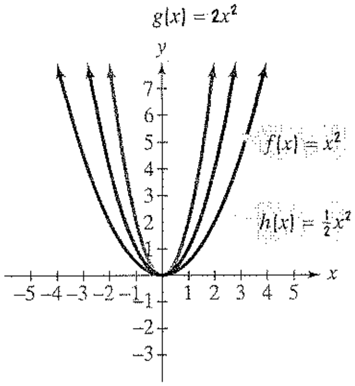


Figure 1.58 Vertically stretching and shrinking $f(x) = x^2$

Vertical Stretching and Shrinking

Morphing does much more than move an image horizontally, vertically, or about an axis. An object having one shape is transformed into a different shape. Horizontal shifts, vertical shifts, and reflections do not change the basic shape of a graph. Graphs remain rigid and proportionally the same when they undergo these transformations. How can we shrink and stretch graphs, thereby altering their basic shapes?

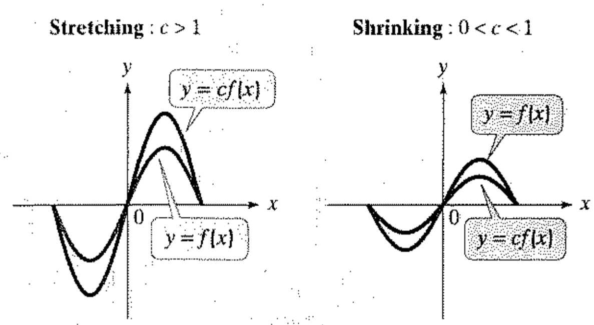
Look at the three graphs in **Figure 1.58**. The black graph in the middle is the graph of the standard quadratic function, $f(x) = x^2$. Now, look at the blue graph on the top. The equation of this graph is $g(x) = 2x^2$, or $g(x) = 2f(x)$. Thus, for each x, the y-coordinate of g is 2 times as large as the corresponding y-coordinate on the graph of f. The result is a narrower graph because the values of y are rising faster. We say that the graph of g is obtained by vertically *stretching* the graph of f. Now, look at the red graph on the bottom. The equation of this graph is $h(x) = \frac{1}{2}x^2$, or $h(x) = \frac{1}{2}f(x)$. Thus, for each x, the y-coordinate of h is one-half as large as the corresponding y-coordinate on the graph of f. The result is a wider graph because the values of y are rising more slowly. We say that the graph of h is obtained by vertically *shrinking* the graph of f.

These observations can be summarized as follows:

Vertically Stretching and Shrinking Graphs

Let f be a function and c a positive real number.

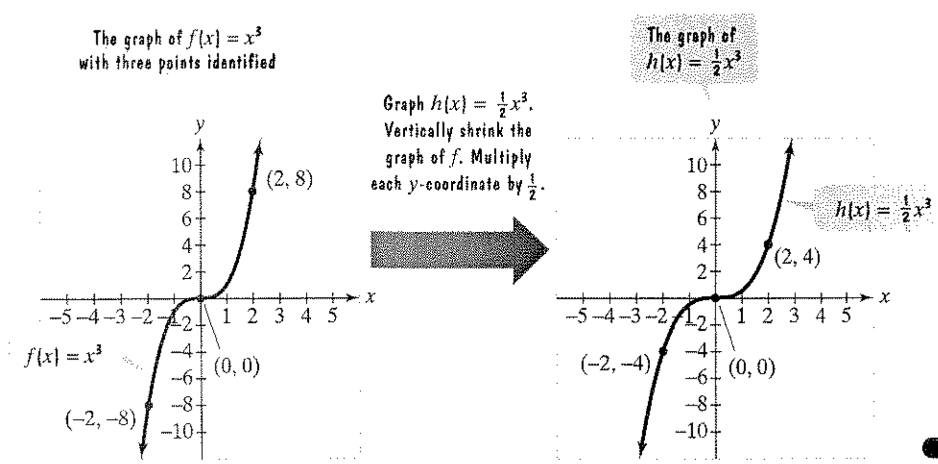
- If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of its y-coordinates by c.
- If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of its y-coordinates by c.



EXAMPLE 6 Vertically Shrinking a Graph

Use the graph of $f(x) = x^3$ to obtain the graph of $h(x) = \frac{1}{2}x^3$.

Solution The graph of $h(x) = \frac{1}{2}x^3$ is obtained by vertically shrinking the graph of $f(x) = x^3$.



A function involving more than one transformation can be graphed by performing transformations in the following order:

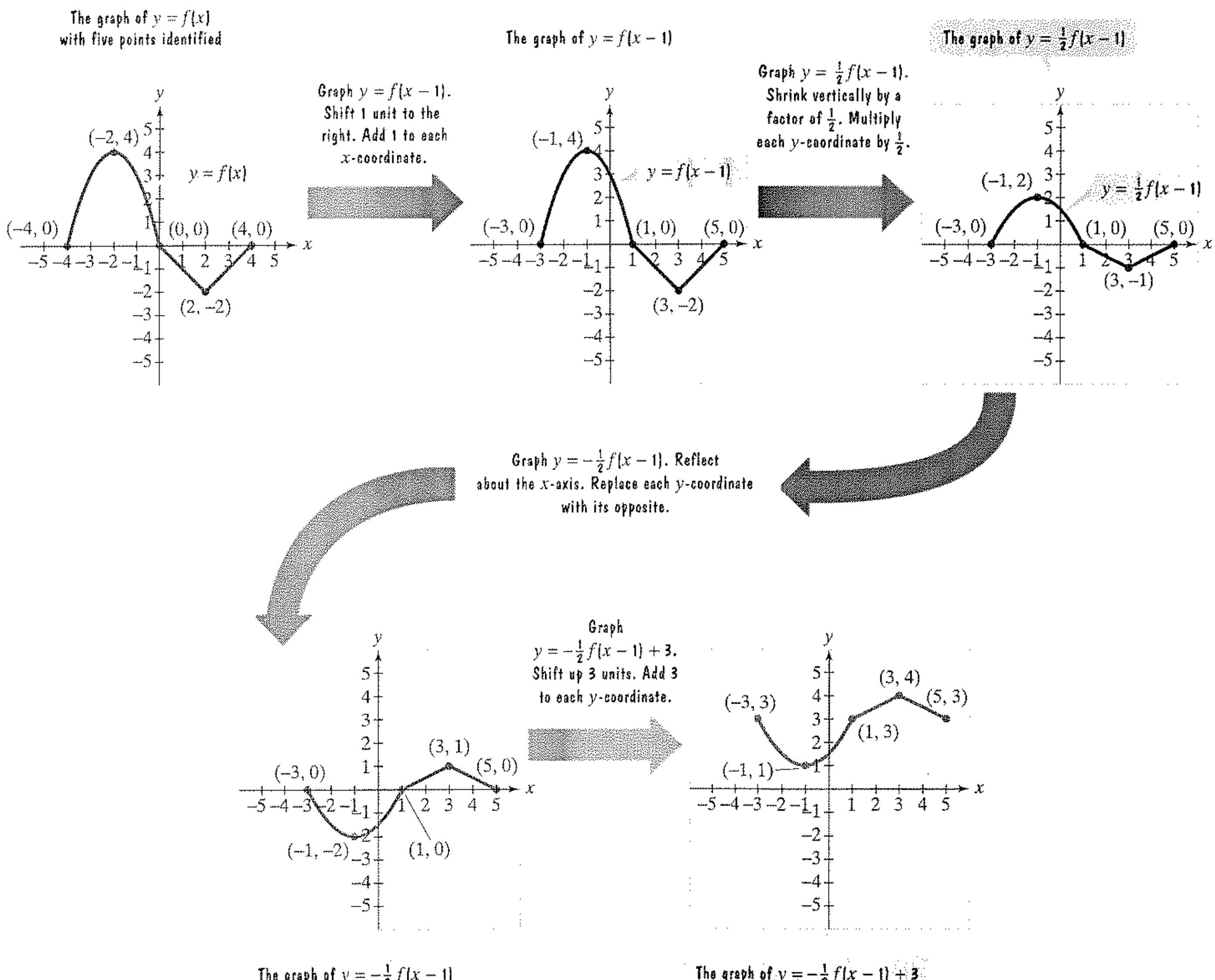
1. Horizontal shifting
2. Stretching or shrinking
3. Reflecting
4. Vertical shifting

EXAMPLE 8 Graphing Using a Sequence of Transformations

Use the graph of $y = f(x)$ given in **Figure 1.59** of Example 7 on page 212, and repeated below, to graph $y = -\frac{1}{2}f(x-1)+3$.

Solution Our graphs will evolve in the following order:

1. Horizontal shifting: Graph $y = f(x-1)$ by shifting the graph of $y = f(x)$ 1 unit to the right.
2. Shrinking: Graph $y = \frac{1}{2}f(x-1)$ by shrinking the previous graph by a factor of $\frac{1}{2}$.
3. Reflecting: Graph $y = -\frac{1}{2}f(x-1)$ by reflecting the previous graph about the x-axis.
4. Vertical shifting: Graph $y = -\frac{1}{2}f(x-1)+3$ by shifting the previous graph up 3 units.



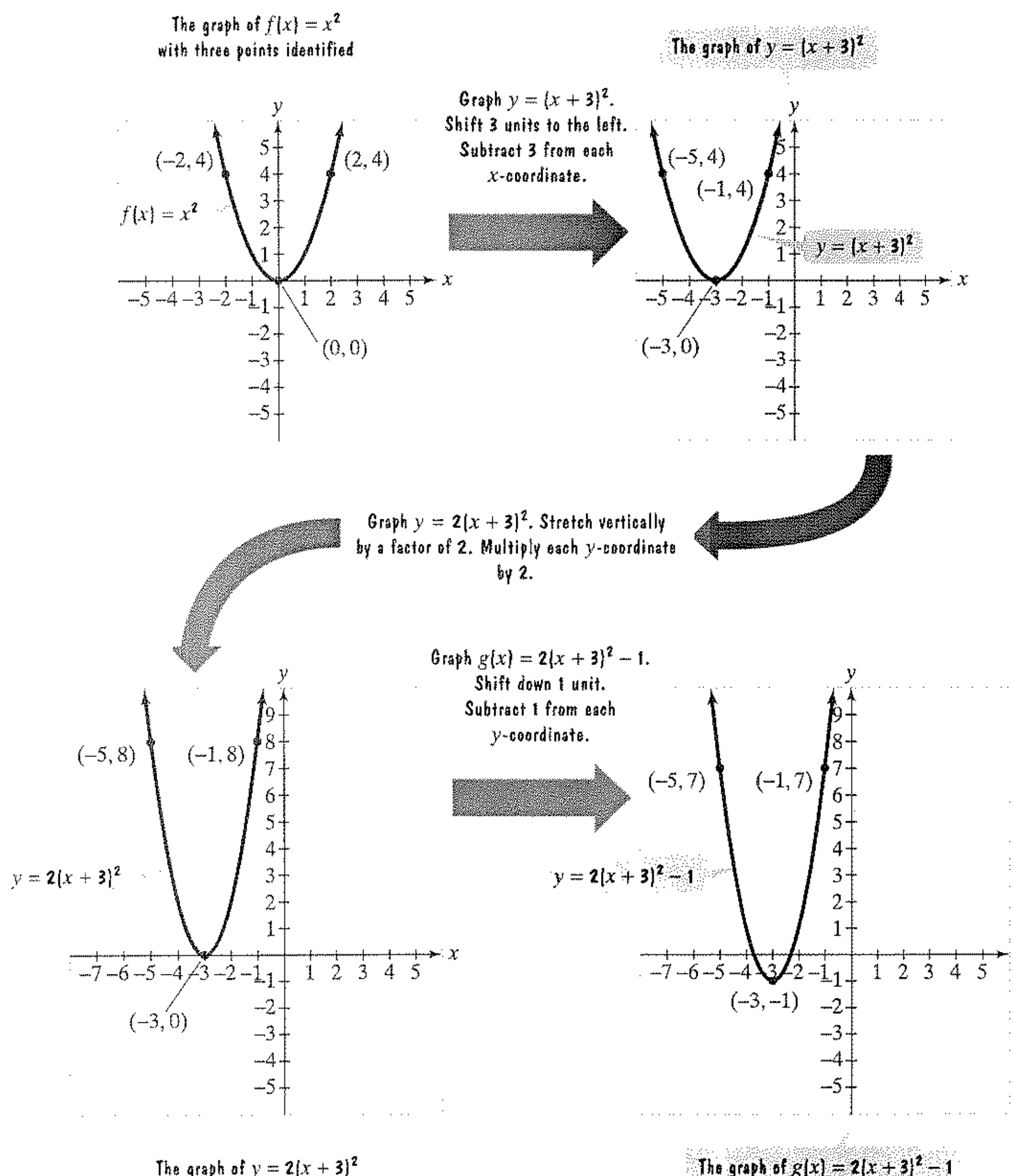
Check Point 8 Use the graph of $y = f(x)$ given in **Figure 1.60** of Check Point 7 on page 213 to graph $y = -\frac{1}{3}f(x + 1) - 2$.

EXAMPLE 9 Graphing Using a Sequence of Transformations

Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x + 3)^2 - 1$.

Solution Our graphs will evolve in the following order:

1. Horizontal shifting: Graph $y = (x + 3)^2$ by shifting the graph of $f(x) = x^2$ three units to the left.
2. Stretching: Graph $y = 2(x + 3)^2$ by stretching the previous graph by a factor of 2.
3. Vertical shifting: Graph $g(x) = 2(x + 3)^2 - 1$ by shifting the previous graph down 1 unit.



Check Point 9 Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x - 1)^2 + 3$.