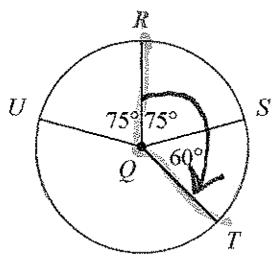
Chapter 11: Circles Name________________________

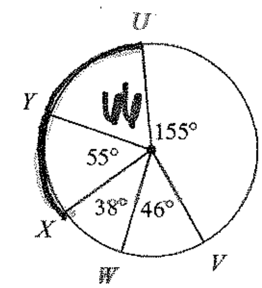
Lesson 11.2 ~ Extra Note Sheet

Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

1) $m\angle RQT$ = 135°



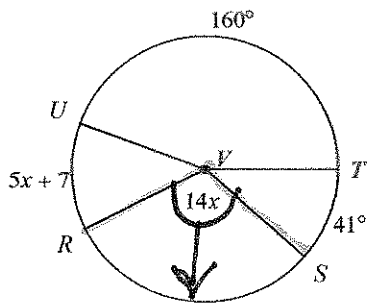
2) $m\overset{\frown}{XU}$ = 121°



360-294=66

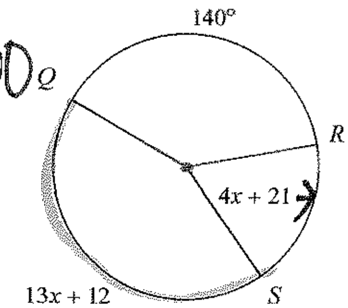
Set up and solve an equation to find the value of x. Then find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

3) $m\angle SVR$ = 112°



19x+208=360
19x=152
x=8

4) $m\overset{\frown}{SQ}$ = 155°

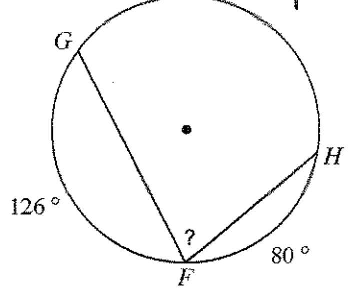


17x+173=360
17x=187
x=11

The Inscribed Angle Theorem states: "The measure of an inscribed angle is one half the measure of its intercepted arc."

Use the Inscribed Angle Theorem to find the measure of the arc or angle indicated.

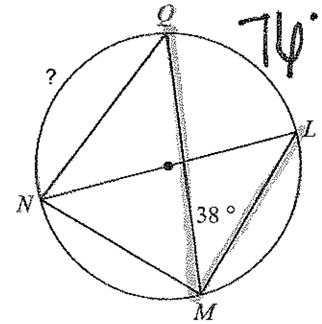
5) 154



∠F = 1/2 · GH

∠F = 77°

6) 74°

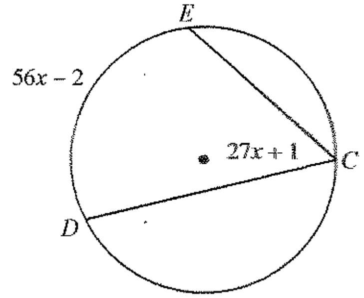


$m\overset{\frown}{NQL}$ = 180

$m\overset{\frown}{NQ}$ = 104°

Use the Inscribed Angle Theorem to set up and solve an equation to find the value of x. Then find the measure of the arc or angle indicated.

7) Find $m\angle DCE$ = 55°



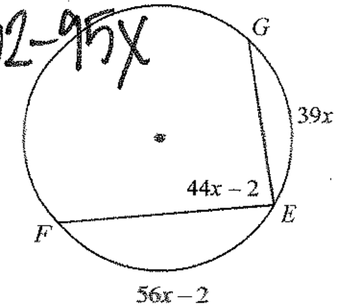
$m\angle DCE = \frac{1}{2} \cdot m\overset{\frown}{DE}$

$27x+1 = \frac{1}{2}(56x-2)$

$27x+1 = 28x-1$

$2 = x$

8) Find $m\overset{\frown}{EG}$ = 78°



362−95X

$\overset{\frown}{FG} = 360 - (95x-2)$

$360 - 95x + 2$

$362 - 95x$

$m\angle FEG = \frac{1}{2} \cdot m\overset{\frown}{FG}$

$2 \cdot m\angle FEG = m\overset{\frown}{FG}$

$2(44x-2) = 362-95x$

$88x - 4 = 362 - 95x$

$183x = 366$

$x = 2$

#8

EASIER METHOD

$m\overset{\frown}{FG} = 2 \cdot m\angle E$

$= 2(44x-2) = 88x-4$

whole circle:

$88x - 4 + 39x + 56x - 2 = 360$

$183x - 6 = 360$

$183x = 366$

$x = 2$

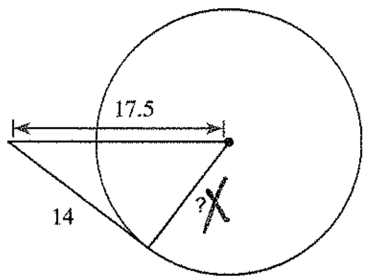
Chapter 10: Circles Name________________________

Lesson 11.3 ~ Extra Note Sheet

The Tangent to a Circle Theorem states: "A line drawn tangent to a circle is perpendicular to a radius of the circle drawn to the point of tangency."

Use the Tangent to a Circle Theorem and the Pythagorean Theorem to find the segment length indicated. Assume that lines which appear to be tangent are tangent.

1)



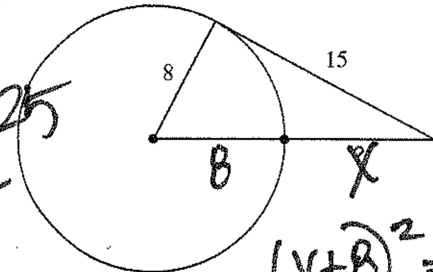
$x^2 + 14^2 = 17.5^2$

$x^2 + 196 = 306.25$

$x^2 = 110.25$

$x = 10.5$

2)



$(x+8)^2 = 8^2 + 15^2$

$x^2 + 16x + 64 = 289$

$x^2 + 16x - 225 = 0$

$(x+25)(x-9) = 0$

$x = 9$

OR

$\sqrt{(x+8)^2} = \sqrt{289}$

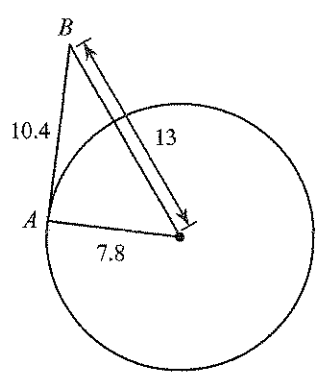
$x + 8 = \pm 17$

$x = 17 - 8 = 9$

$x = -17 - 8 = -2$

Use the Tangent to a Circle Theorem and the Pythagorean Theorem to determine if line *AB* is tangent to the circle.

3)



$7.8^2 + 10.4^2 \stackrel{?}{=} 13^2$

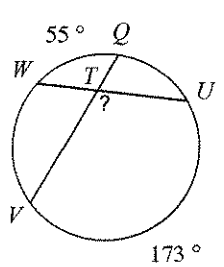
$169 = 169$ ✱

TANGENT

The Interior Angles of a Circle Theorem states: "If an angle is formed by two intersecting chords or secants such that the vertex of the angle is in the interior of the circle, then the measure of the angle is half the sum of the measures of the arcs intercepted by the angle and its vertical angle."

Use the Interior Angles of a Circle Theorem to find the measure of the arc or angle indicated. Assume that lines which appear tangent are tangent.

4)

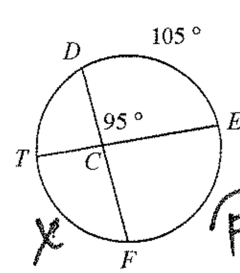


$\angle VTU = \frac{1}{2}(55 + 173)$

$\angle VTU = \frac{1}{2}(228)$

$\angle VTU = 114°$

5)



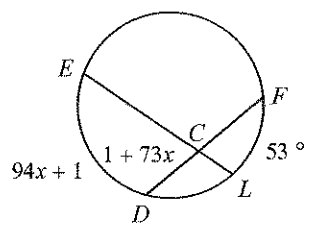
$2 \cdot 95 = \cancel{\frac{1}{2}}(x + 105)$

$190 = x + 105$

$\overset{\frown}{FT} = 85 = x$

Use the Interior Angles of a Circle Theorem to set up and solve an equation to find the value of *x*. Then find the measure of the arc or angle indicated. Assume that lines which appear tangent are tangent.

6) Find $m\angle DCE$ = 74°



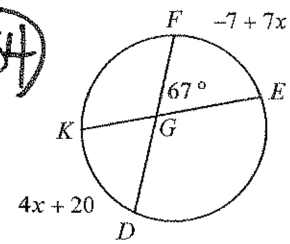
$1+73x = \frac{1}{2}(94x+54)$

$1+73x = 47x+27$

$26x = 26$

$x = 1$

7) Find $m\overparen{FE}$ = 70°



$67 = \frac{1}{2}(11x+13)$

$134 = 11x+13$

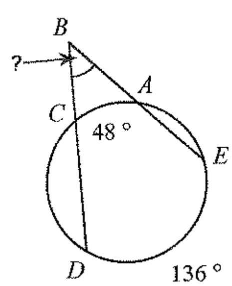
$121 = 11x$

$11 = x$

The Exterior Angles of a Circle Theorem states: "If an angle is formed by two intersecting secants, two intersecting tangents, or an intersecting tangent and secant such that the vertex of the angle is in the exterior of a circle, then the measure of the angle is half the difference of the measure of the arc(s) intercepted by the angle."

Use the Exterior Angles of a Circle Theorem to find the measure of the arc or angle indicated. Assume that lines which appear tangent are tangent.

8)

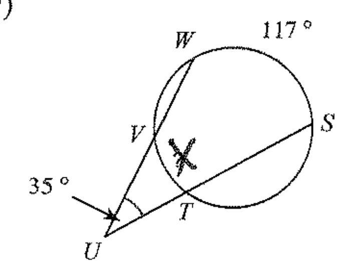


$\angle B = \frac{1}{2}(136-48)$

$\angle B = \frac{1}{2}(88)$

$\angle B = 44°$

9)



$35 = \frac{1}{2}(117-x)$

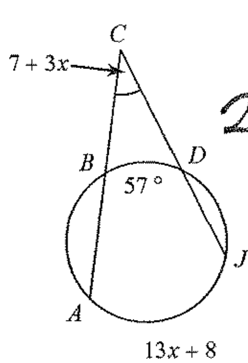
$70 = 117-x$

$+47 = +x$

$VT = 47°$

Use the Exterior Angles of a Circle Theorem to set up and solve an equation to find the value of *x*. Then find the measure of the arc or angle indicated. Assume that lines which appear tangent are tangent.

10) Find $m\angle ACJ$ → 34°



$7+3x = \frac{1}{2}(13x+8-57)$

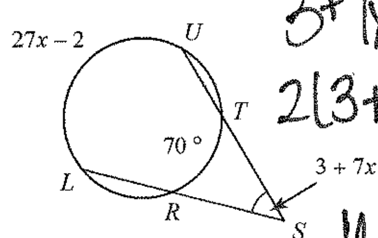
$2(7+3x) = 13x-49$

$14+6x = 13x-49$

$63 = 7x$

$9 = x$

11) Find $m\overparen{LU}$ • 160°



$3+7x = \frac{1}{2}(27x-2-70)$

$2(3+7x) = 27x-72$

$6+14x = 27x-72$

$78 = 13x$

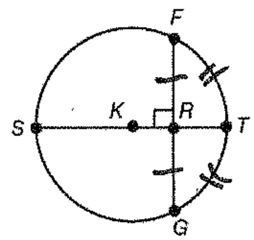
$6 = x$

Lesson 11.4 ~ Extra Note Sheet

The Diameter-Chord Theorem states: "If a circle's diameter is perpendicular to a chord, then the diameter bisects the chord and bisects the arc determined by the chord."

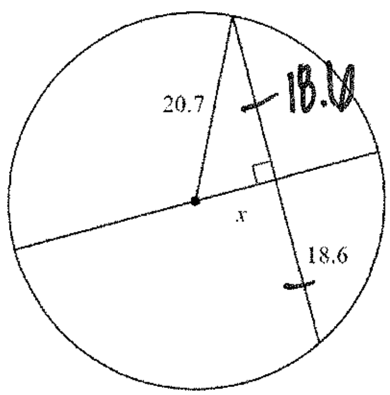
Example

In circle K, diameter $\overline{ST}$ is perpendicular to chord $\overline{FG}$. So $FR = GR$ and $m\overset{\frown}{FT} = m\overset{\frown}{GT}$.



Use the Diameter-Chord Theorem and the Pythagorean Theorem to find the value of x.

1.

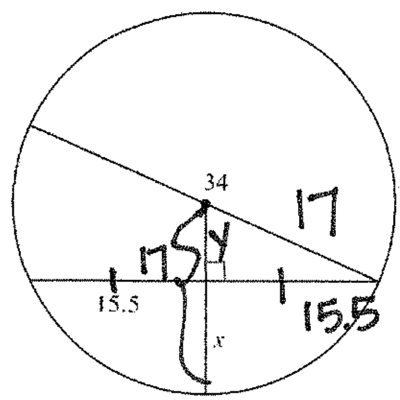


$x^2 + 18.6^2 = 20.7^2$

$x^2 + 345.96 = 428.49$

$x^2 = 82.53$ $x \approx 9.1$

2.



$x = 17 - 7$

$x \approx 10$

$y^2 + 15.5^2 = 17^2$

$y^2 + 240.25 = 289$

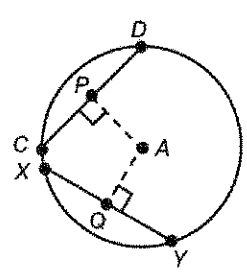
$y^2 = 48.75$

$y \approx 7.0$

The Equidistant Chord Theorem states: "If two chords of the same circle or congruent circles are congruent, then they are equidistant from the center of the circle."

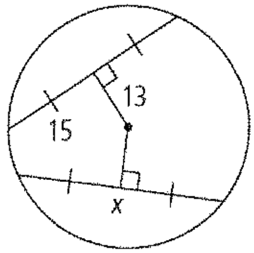
Example

In circle A, chord $\overline{CD}$ is congruent to chord $\overline{XY}$. So $PA = QA$.



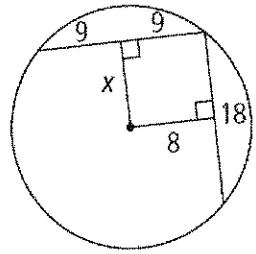
Use the Equidistant Chord Theorem to find the value of x.

3.



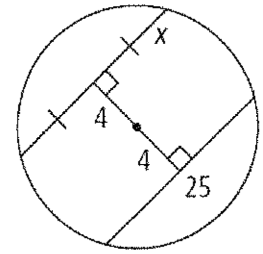
$X=30$

4.



$X=8$

5.

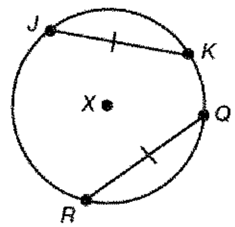


$X=12.5$

The Congruent Chord – Congruent Arc Theorem states: "If two chords of the same circle or congruent circles are congruent, then their corresponding arcs are congruent."

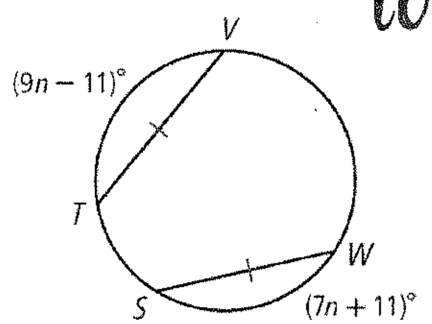
Example

In circle X, chord $\overline{JK}$ is congruent to chord $\overline{QR}$. So $m\widehat{JK} = m\widehat{QR}$.



Use the Congruent Chord – Congruent Arc Theorem to set up and solve an equation to find the value of x. Then find the indicated arc measure.

6. $\overline{TV} \cong \overline{WS}$. Find $m\widehat{WS}$. $= 88^\circ$

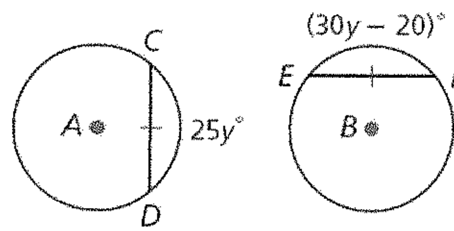


$9n-11=7n+11$

$2n=22$

$n=11$

7. $\odot A \approx \odot B$. $\overline{CD} \cong \overline{EF}$. Find $m\widehat{CD}$. $= 100^\circ$



$25y=30y-20$

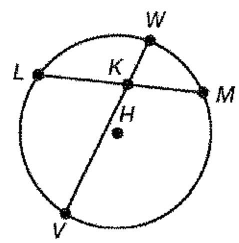
$-5y=-20$

$y=4$

The Segment-Chord Theorem states: "If two chords in a circle intersect, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the second chord."

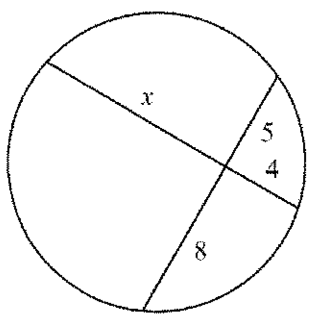
Example

In circle H, chords $\overline{LM}$ and $\overline{VW}$ intersect to form $\overline{LK}$ and $\overline{MK}$ of chord $\overline{LM}$ and $\overline{WK}$ and $\overline{VK}$ of chord $\overline{VW}$. So $LK \cdot MK = WK \cdot VK$.



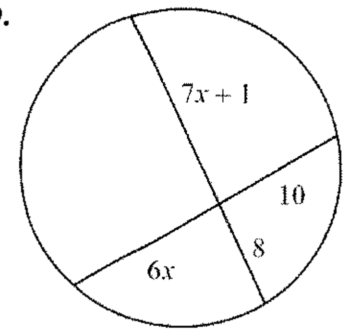
Use the Segment-Chord Theorem to set up and solve an equation to find the value of x.

8.



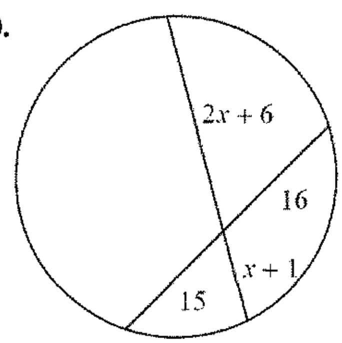
$4x = 5 \cdot 8$
$4x = 40$
$x = 10$

9.



$8(7x+1) = 6x \cdot 10$
$56x + 8 = 60x$
$8 = 4x$
$2 = x$

10.



$(2x+6)(x+1) = 15 \cdot 16$
$2x^2 + 2x + 6x + 6 = 240$
$2x^2 + 8x - 234 = 0$
$2(x^2 + 4x - 117) = 0$
$2(x+13)(x-9) = 0$
$x = -13$ or $x = 9$

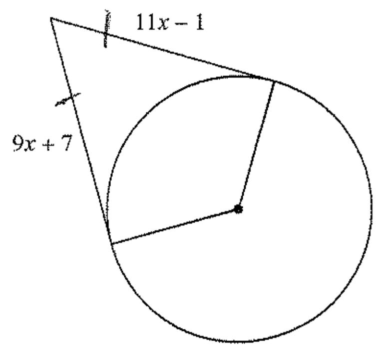
Name______________________________

Lesson 11.5 ~ Extra Note Sheet

The Tangent Segment Theorem states: "If two segments are drawn from the same point on the exterior of a circle, then the tangent segments are congruent."

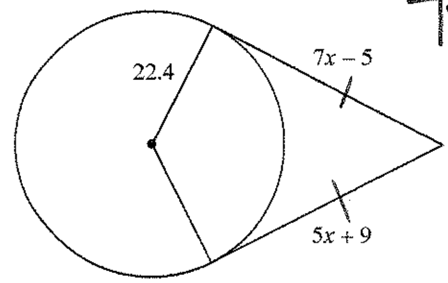
Use the Tangent Segment Theorem to set up and solve and equation to find the value of *x*. Assume that lines which appear to be tangent are tangent.

1)



$11x-1=9x+7$
$2x=8$
$x=4$

2)

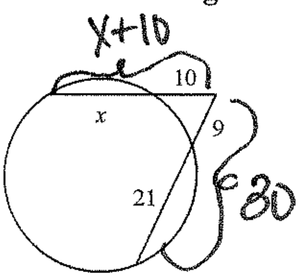


$7x-5=5x+9$
$2x=14$
$x=7$

The Secant Segment Theorem states: "If two secant segments intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external secant segment is equal to the product of the lengths of the second secant segment and its external secant segment."

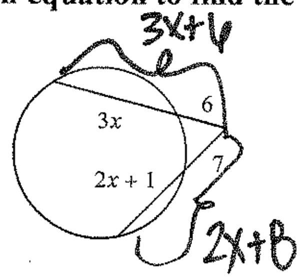
Use the Secant Segment Theorem to set up and solve an equation to find the value of *x*.

3)



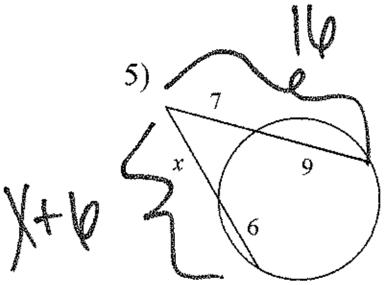
$10(x+10)=9\cdot 30$
$10x+100=270$
$10x=170$
$x=17$

4)



$6(3x+6)=7(2x+8)$
$18x+36=14x+56$
$4x=20$
$x=5$

5)

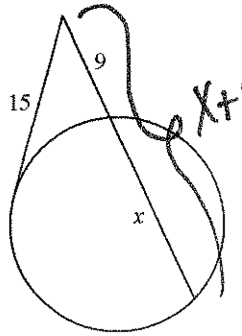


$x(x+6)=7\cdot 16$
$x^2+6x=112$
$x^2+6x-112=0$
$(x+14)(x-8)=0$
$x=-14$ or $x=8$

The Secant Tangent Theorem states: "If a tangent and a secant segment intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the square of the length of the tangent segment."

Use the Secant Tangent Theorem to set up and solve an equation to find the value of x. Assume that lines which appear tangent are tangent.

6)



$X+9$

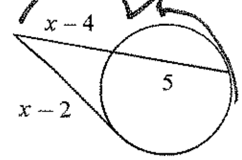
$15^2 = 9(X+9)$

$225 = 9X+81$

$144 = 9X$

$16 = X$

7)



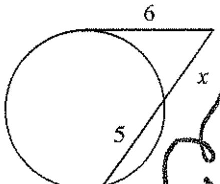
$X+1$

$(X-2)^2 = (X-4)(X+1)$

$X^2-4X+4 = X^2-3X-4$

$+X = +8$

8)



$X+5$

$6^2 = X(X+5)$

$36 = X^2+5X$

$0 = X^2+5X-36$

$0 = (X+9)(X-4)$

$X=-9$ OR $X=4$

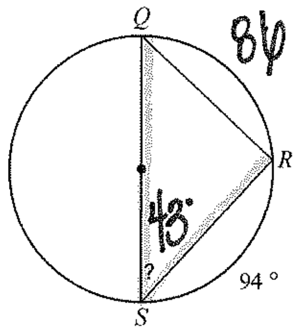
 Name________________________________

Lesson 12.1 ~ Extra Note Sheet

The Inscribed Right Triangle-Diameter Theorem states: "If a triangle is inscribed in a circle such that one side of the triangle is a diameter of the circle, then the triangle is a right triangle.

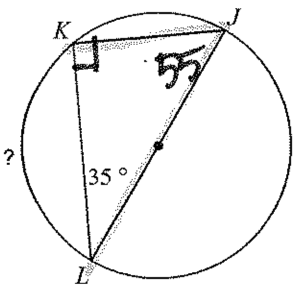
Use the Inscribed Right Triangle-Diameter Theorem to find the measure of the arc or angle indicated.

1)



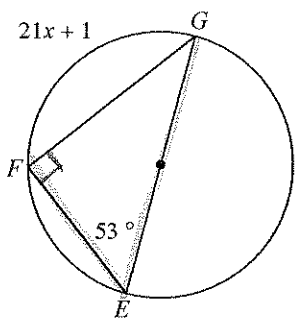
2)

110°



Use the Inscribed Right Triangle-Diameter Theorem to set up and solve an equation to find the value of *x*.

3)



$2 \cdot 53 = 21x + 1$

$106 = 21x + 1$

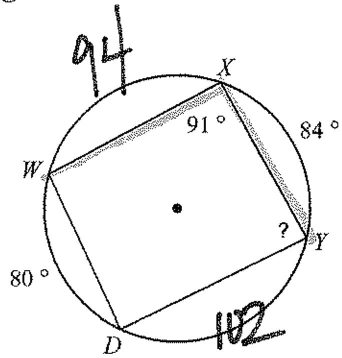
$105 = 21x$

$5 = x$

The Inscribed Quadrilateral-Opposite Angles Theorem states: "If a quadrilateral in inscribed in a circle, then the opposite angles are supplementary."

Use the Inscribed Quadrilateral-Opposite Angles Theorem to find the measure of the arc or angle indicated.

4)



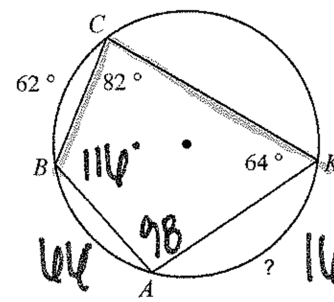
$\angle y = \frac{1}{2}(94 + 80)$

$\angle y = \frac{1}{2}(174)$

$\angle y = 87°$

$m\overset{\frown}{WDY} = 182$

5)

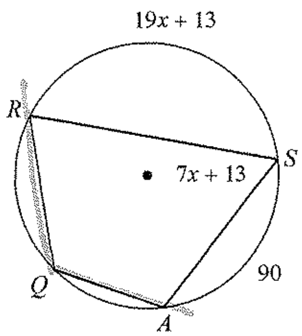


$164 - 66 = 98°$

$m\overset{\frown}{CBA} = 128°$ $m\overset{\frown}{BAK} = 164$

Use the Inscribed Quadrilateral-Opposite Angles Theorem to set up and solve an equation to find the value of x.

6)



$\angle RQA = 180 - (7x + 13) = 167 - 7x$

$\angle RQA = \frac{1}{2} \cdot m\overset{\frown}{RSA}$

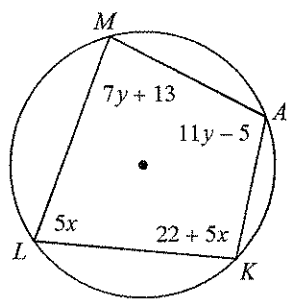
$2(167 - 7x) = 19x + 103$

$334 - 14x = 19x + 103$

$231 = 33x \qquad x = 7$

Use the Inscribed Quadrilateral-Opposite Angles Theorem to set up and solve a system of equations to find the value of x and y.

7)



$5x + 11y - 5 = 180 \longrightarrow 5x + 11y = 185$

$7y + 13 + 22 + 5x = 180 \rightarrow - \; 5x + 7y = 145$

$4y = 40$

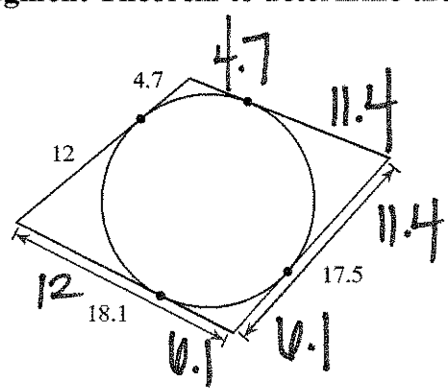
$y = 10$

$x = 15$

The Tangent Segment Theorem states: "If two segments are drawn from the same point on the exterior of a circle, then the tangent segments are congruent."

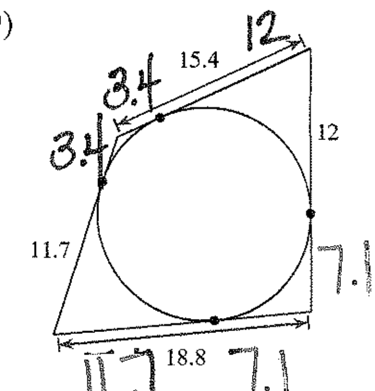
Consider the quadrilateral shown, which is circumscribed about a circle. Use the Tangent Segment Theorem to determine the perimeter of each quadrilateral.

8)



$P = 18.1 + 17.5 + 16.7 + 16.1 = 68.4$

9)



$P = 15.4$
18.8
19.1
$+\, 15.1$
68.4

Lesson 13.2 ~ The Equation for a Circle

The Standard Form of the Equation of a Circle

1) The standard form of the equation of a circle centered at (h, k) and a radius of length r is...

$$(x-h)^2 + (y-k)^2 = r^2$$

Write the standard form equation of the circle with the given center and radius.

2) Center: $(7, -1)$
Radius: 6

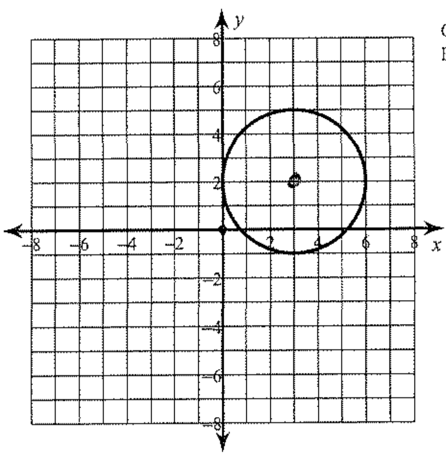
$(x-7)^2 + (y+1)^2 = 36$

3) Center: $(1, 14)$
Radius: $3\sqrt{2}$

$(x-1)^2 + (y-14)^2 = 18$

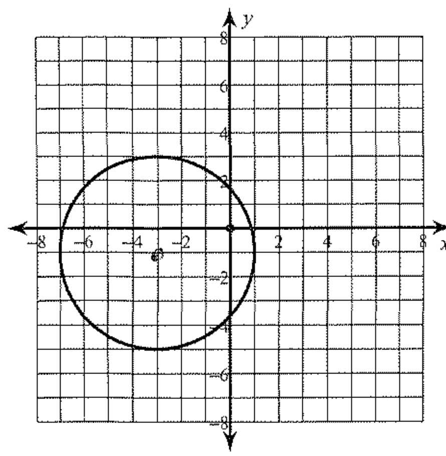
Given the standard form of the equation of a circle, identify the center and radius and then sketch the graph.

4) $(x-3)^2 + (y-2)^2 = 9$



Center: $(3, 2)$
Radius: 3

5) $(x+3)^2 + (y+1)^2 = 16$



Center: $(-3, -1)$
Radius: 4

Write the standard form equation of the circle described.

6) Center: $(\overset{h}{-5}, \overset{k}{14})$
Point on Circle: $(-4, 10)$

$(x+5)^2 + (y-14)^2 = 17$

Find the radius.

$r = \sqrt{(-5+4)^2 + (14-10)^2}$

$r = \sqrt{(-1)^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$

$(x-(-5))^2 + (y-14)^2 = (\sqrt{17})^2$

$(x+5)^2 + (y-14)^2 = 17$

Find the center and the radius of the circle described. Then write the standard form equation of the circle.

7) Ends of a diameter: $(3, -3)$ and $(-11, 15)$

$(x+4)^2 + (y-6)^2 = 130$

Find center (midpt)

$(h,k) = \left(\frac{3+(-11)}{2}, \frac{-3+15}{2}\right)$ (-8, 12)

$(h,k) = (-4, 6)$

Find radius:

$r = \sqrt{(3+4)^2 + (-3-6)^2}$ (7, -9)

$r = \sqrt{49+81} = \sqrt{130}$

$(x+4)^2 + (y-6)^2 = 130$

BONUS: Complete the square (twice) to transform the equation of a circle in general form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ into standard form. Then identify its center and its radius.

8) $x^2 + y^2 - 20x + 8y + 100 = 0$

$(x-10)^2 + (y+4)^2 = 16$

$x^2 - 20x + \underline{100} + y^2 + 8y + \underline{16} = -100 + \underline{100} + \underline{16}$

$(x-10)^2 + (y+4)^2 = 16$

center $(10, -4)$

radius $= 4$

9) $x^2 + y^2 - 22x - 12y + 148 = 0$

$(x-11)^2 + (y-6)^2 = 9$

$x^2 - 22x + \underline{121} + y^2 - 12y + \underline{36} = -148 + \underline{121} + \underline{36}$

$(x-11)^2 + (y-6)^2 = 9$

center: $(11, 6)$

radius $= 3$

10) $x^2 + y^2 - 14x + 13 = 0$

$(x-7)^2 + y^2 = 36$

$x^2 - 14x + \underline{49} + y^2 = -13 + \underline{49}$

$(x-7)^2 + y^2 = 36$

center: $(7, 0)$

radius $= 6$

11) $x^2 + y^2 - 6x - 12y + 20 = 0$

$(x-3)^2 + (y-6)^2 = 25$

$x^2 - 6x + \underline{9} + y^2 - 12y + \underline{36} = -20 + \underline{9} + \underline{36}$

$(x-3)^2 + (y-6)^2 = 25$

center: $(3, 6)$

radius $= 5$