Factoring expressions and Solving Equations That Are Quadratic in Form

You have already learned how to factor quadratic expressions in the form of $ax^2 + bx + c$ where $a \neq 0$. However, there are instances where you will be faced with trinomials of a degree higher than two but still fit the quadratic form. For example, the trinomial $ax^4 + bx^2 + c$ has a degree power of four but it still fits the quadratic form.

$$ax^4 + bx^2 + c = a(x^2)^2 + b(x^2) + c$$
$$= au^2 + bu + c$$

This substitution pattern can also be applied to situations where there is an algebraic expression in place of x in the quadratic form. Take for example the expression $2(x-3)^2 - 5(x-3) - 12$. In this trinomial instead of have x and x^2 we have $(x-3)$ and $(x-3)^2$.

$$2(x-3)^2 - 5(x-3) - 12 = 2(x-3)^2 - 5(x-3) - 12$$
$$= 2u^2 - 5u - 12$$

The following steps can be used to solve equations that are quadratic in form:

1. Let u equal a function of the original variable (normally the middle term)
2. Substitute u into the original equation so that it is in the form $au^2 + bu + c = 0$
3. Factor the quadratic equation using the methods learned earlier
4. Solve the equation for u
5. Replace u with the expression of the original variable
6. Solve the resulting equation for the original variable
7. Check for any extraneous solutions

Example 1: Solve the equation $x^4 - 13x^2 + 36 = 0$.

Solution

Step 1: Let u equal a function of the original variable

In this problem, we would let u equal x^2

Step 2: Substitute u into the original equation for the variable expression

Before performing the substitution rewrite x^4 as a multiply of x^2 which will be replaced by u. $x^4 = (x^2)^2$

$$x^4 - 13x^2 + 36 = 0$$
$$(x^2)^2 - 13x^2 + 36 = 0$$
$$u^2 - 13u + 36 = 0$$

Example 1 (Continued):

Step 3: Factor the quadratic equation

$u^2 - 13u + 36 = 0$
$(u - 4)(u - 9) = 0$

Step 4: Solve the equation for u

$(u - 4)(u - 9) = 0$
$u - 4 = 0$ or $u - 9 = 0$
$u = 4$ or $u = 9$

Step 5: Replace u with the expression of the original variable

$u = 4$ or $u = 9$
$x^2 = 4$ or $x^2 = 9$

Step 6: Solve for the original variable

$x^2 = 4$ or $x^2 = 9$
$x^2 - 4 = 0$ or $x^2 - 9 = 0$
$(x - 2)(x + 2) = 0$ or $(x - 3)(x + 3) = 0$
$x - 2 = 0$ or $x + 2 = 0$ or $x - 3 = 0$ or $x + 3 = 0$
$x = 2$ or $x = -2$ or $x = 3$ or $x = -3$

Step 7: Check for any extraneous solutions

$x = 2$

$x^4 - 13x^2 + 36 = 0$
$(2)^4 - 13(2)^2 + 36 = 0$
$16 - 52 + 36 = 0$
$52 - 52 = 0$
$0 = 0$

$x = -2$

$x^4 - 13x^2 + 36 = 0$
$(-2)^4 - 13(-2)^2 + 36 = 0$
$16 - 52 + 36 = 0$
$52 - 52 = 0$
$0 = 0$

Example 1 (Continued):

$x = 3$

$x^4 – 13x^2 + 36 = 0$
$(3)^4 – 13(3)^2 + 36 = 0$
$81 – 117 + 36 = 0$
$117 – 117 = 0$
$0 = 0$

$x = -3$

$x^4 – 13x^2 + 36 = 0$
$(-3)^4 – 13(-3)^2 + 36 = 0$
$81 – 117 + 36 = 0$
$117 – 117 = 0$
$0 = 0$

Example 2: Solve the equation $2x^{2/3} – 7x^{1/3} + 6 = 0$.

Solution

Step 1: Let u equal a function of the original variable

In this problem, we would let u equal $x^{1/3}$

Step 2: Substitute u into the original equation for the variable expression

Before performing the substitution rewrite $x^{2/3}$ as a multiply of $x^{1/3}$ which will be replaced by u. $x^{2/3} = (x^{1/3})^2$

$2x^{2/3} – 7x^{1/3} + 6 = 0$
$2(x^{1/3})^2 – 7x^{1/3} + 6 = 0$
$2u^2 – 7u + 6 = 0$

Step 3: Factor the quadratic equation

$2u^2 – 7u + 6 = 0$
$(2u – 3)(u – 2) = 0$

Step 4: Solve the equation for u

$(2u – 3)(u – 2) = 0$
$2u – 3 = 0$ or $u – 2 = 0$
$2u = 3$ or $u = 2$

$u = 3/2$

Example 1 (Continued):

Step 5: Replace u with the expression of the original variable

$u = 3/2$ or $u = 2$
$x^{1/3} = 3/2$ or $x^{1/3} = 2$

Step 6: Solve for the original variable

$x^{1/3} = 3/2$ or $x^{1/3} = 2$
$(x^{1/3})^3 = (3/2)^3$ or $(x^{1/3})^3 = (2)^3$
$x = 27/8$ or $x = 8$

Step 7: Check for any extraneous solutions

$x = 27/8$

$2x^{2/3} - 7x^{1/3} + 6 = 0$
$2(27/8)^{2/3} - 7(27/8)^{1/3} + 6 = 0$
$2[(27/8)^{1/3}]^2 - 7(27/8)^{1/3} + 6 = 0$
$2(3/2)^2 - 7(3/2) + 6 = 0$
$2(9/4) - 21/2 + 6 = 0$
$9/2 - 21/2 + 6 = 0$
$- 12/2 + 6 = 0$
$- 6 + 6 = 0$
$0 = 0$

$x = 8$

$2x^{2/3} - 7x^{1/3} + 6 = 0$
$2(8)^{2/3} - 7(8)^{1/3} + 6 = 0$
$2[(8)^{1/3}]^2 - 7(8)^{1/3} + 6 = 0$
$2(2)^2 - 7(2) + 6 = 0$
$2(4) - 14 + 6 = 0$
$8 - 14 + 6 = 0$
$14 - 14 = 0$
$0 = 0$