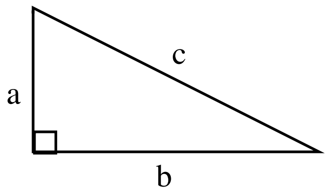
SOLVING RIGHT TRIANGLES

PYTHAGOREAN THEOREM



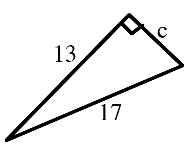
Any triangle that has a right angle is called a **RIGHT TRIANGLE**. The two sides that form the right angle, a and b, are called **LEGS**, and the side opposite (that is, across the triangle from) the right angle, c, is called the **HYPOTENUSE**.

For any right triangle, the sum of the squares of the legs of the triangle is equal to the square of the hypotenuse, that is, $a^2 + b^2 = c^2$. This relationship is known as the **PYTHAGOREAN THEOREM**. In words: $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$.

Extension: If $(\text{leg})^2 + (\text{leg})^2 < (\text{hypotenuse})^2$, then the triangle is obtuse and if $(\text{leg})^2 + (\text{leg})^2 > (\text{hypotenuse})^2$, then the triangle is acute.

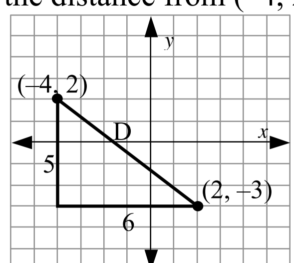
Examples: Draw a diagram, then use the Pythagorean theorem to solve each problem.

a) Solve for the missing side.



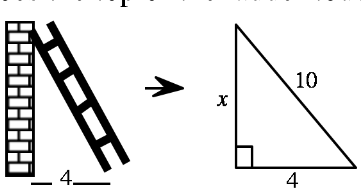
$$c^2 + 13^2 = 17^2$$
$$c^2 + 169 = 289$$
$$c^2 = 120$$
$$c = \sqrt{120}$$
$$c = 2\sqrt{30}$$
$$c \approx 10.95$$

b) Find the distance from (–4, 2) to (2, –3).



After calculating or counting the length of the legs, $D^2 = 5^2 + 6^2 = 61$ and so $D = \sqrt{61} \approx 7.8$

c) One end of a ten foot ladder is four feet from the base of a wall. How high on the wall does the top of the ladder touch?



$$x^2 + 4^2 = 10^2$$
$$x^2 + 16 = 100$$
$$x^2 = 84$$
$$x \approx 9.2$$

The ladder touches the wall about 9.2 feet above the ground.

d) Could 3, 6 and 8 represent the lengths of the sides of a right triangle? Explain.

$$3^2 + 6^2 \stackrel{?}{=} 8^2$$
$$9 + 36 \stackrel{?}{=} 64$$
$$45 \neq 64$$

Since the Pythagorean theorem relationship is not true for these lengths, they cannot be the side lengths of a right triangle.
Furthermore, since 45 < 64, this triangle is obtuse.

Use the Pythagorean theorem to find the value of x. Round answers to the nearest tenth.

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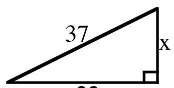

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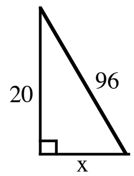

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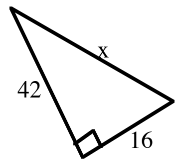

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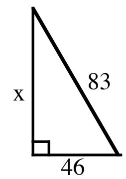

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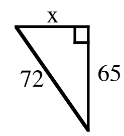

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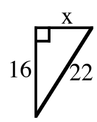

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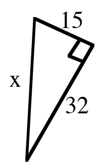

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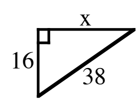

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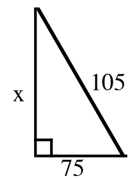

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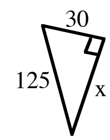


Solve the following word problems. Remember to draw a diagram of each situation.

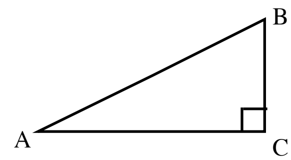
11. A 12 foot ladder is six feet from a wall. How high on the wall does the ladder touch?
12. A 15 foot ladder is five feet from a wall. How high on the wall does the ladder touch?
13. A 9 foot ladder is three feet from a wall. How high on the wall does the ladder touch?
14. What is the distance from (–1, 1) to (3, 4)?
15. What is the distance from (–1, 3) to (4, 1)?
16. What is the distance from (2, 5) to (–3, –1)?
17. Could 8, 12, and 13 represent the lengths of sides of a right triangle? Justify your answer.
18. Could 5, 12, and 13 represent the lengths of sides of a right triangle? Justify your answer.
19. Could 9, 12, and 15 represent the lengths of sides of a right triangle? Justify your answer.
20. Could 10, 15, and 20 represent the lengths of sides of a right triangle? Justify your answer.
21. What is the longest fishing pole that could fit in a 2 foot by 3 foot by 4 foot box?
22. What is the longest straight wire that can be stretched in a 30 foot by 30 foot by 10 foot classroom?

Answers

1. 29.7	2. 93.9	3. 44.9	4. 69.1	5. 31.0
6. 15.1	7. 35.3	8. 34.5	9. 73.5	10. 121.3
11. 10.4 ft	12. 14.1 ft	13. 8.5 ft	14. 5	15. 5.4
16. 7.8	17. no (acute)	18. yes	19. yes	20. no (obtuse)
21. 5.4 ft	22. 43.6 ft			

RIGHT TRIANGLE TRIGONOMETRY

The three basic trigonometric ratios for right triangles are the **sine** (pronounced "sign"), **cosine**, and **tangent**. Each one is used in separate situations, and the easiest way to remember which to use when is the mnemonic SOH-CAH-TOA. With reference to one of the acute angles in a right triangle, <u>S</u>ine uses the <u>O</u>pposite and the <u>H</u>ypotenuse - SOH. The <u>C</u>osine uses the <u>A</u>djacent side and the <u>H</u>ypotenuse - CAH, and the <u>T</u>angent uses the <u>O</u>pposite side and the <u>A</u>djacent side -TOA. In each case, the position of the angle determines which leg (side) is opposite or adjacent. Remember that opposite means “across from” and adjacent means “next to.”



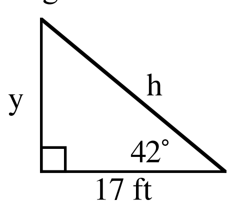
$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC}$$

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB}$$

Example 1

Use trigonometric ratios to find the lengths of each of the missing sides of the triangle below.



The length of the adjacent side with respect to the 42˚ angle is 17 ft. To find the length y, use the tangent because y is the opposite side and we know the adjacent side.

$$\tan 42° = \tfrac{y}{17}$$
$$17\tan 42° = y$$
$$15.307 \text{ ft} \approx y$$

The length of y is approximately 15.31 feet.

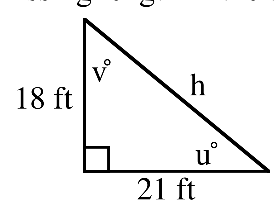
To find the length h, use the cosine ratio (adjacent and hypotenuse).

$$\cos 42° = \tfrac{17}{h}$$
$$h\cos 42° = 17$$
$$h = \tfrac{17}{\cos 42°} \approx 22.876 \text{ ft}$$

The hypotenuse is approximately 22.9 ft long.

Example 2

Use trigonometric ratios to find the size of each angle and the missing length in the triangle below.



To find $m\angle u$, use the tangent ratio because you know the opposite (18 ft) and the adjacent (21 ft) sides.

$$\tan u° = \tfrac{18}{21}$$
$$m\angle u = \tan^{-1}\tfrac{18}{21} \approx 40.601°$$

The measure of angle u is approximately $40.6°$. By subtraction we know that $m\angle v \approx 49.4°$.

Use the sine ratio for $m\angle u$ and the opposite side and hypotenuse.

$$\sin 40.6° = \tfrac{18}{h}$$
$$h\sin 40.6° = 18$$
$$h = \tfrac{\sin 40.6°}{18} \approx 27.659 \text{ ft}$$

The hypotenuse is approximately 27.7 ft long.

Problems

Use trigonometric ratios to solve for the variable in each figure below.

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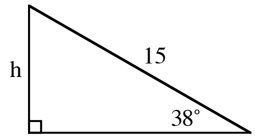

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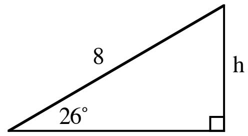

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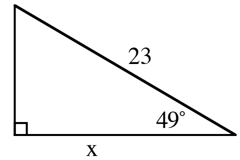

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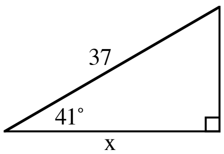

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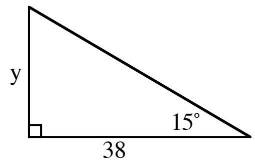

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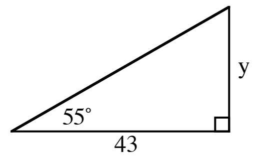

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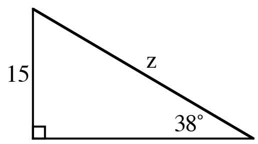

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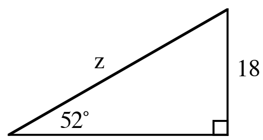

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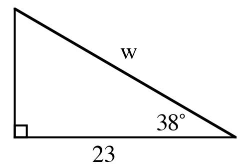

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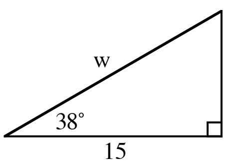

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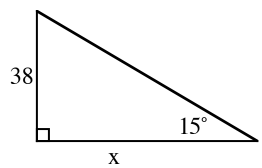

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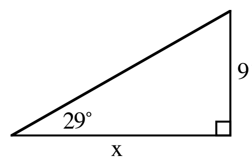

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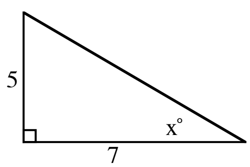

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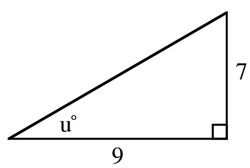

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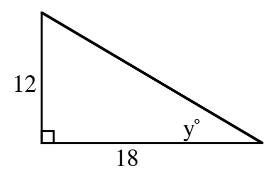

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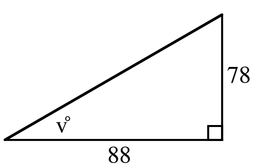


Draw a diagram and use trigonometric ratios to solve each of the following problems.

17. Juanito is flying a kite at the park and realizes that all 500 feet of string are out. Margie measures the angle of the string with the ground by using her clinometer and finds it to be 42.5˚. How high is Juanito's kite above the ground?

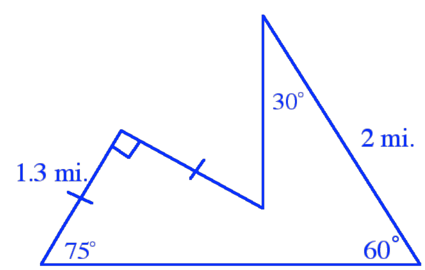
18. Nell's kite has a 350 foot string. When it is completely out, Ian measures the angle with the ground to be 47.5˚. How far would Ian need to walk to be directly under the kite?

19. Mayfield High School's flagpole is 15 feet high. Using a clinometer, Tamara measured an angle of 11.3˚ to the top of the pole. Tamara is 62 inches tall. How far from the flagpole is Tamara standing?

20. Tamara took another sighting of the top of the flagpole from a different position. This time the angle is 58.4˚. If everything else is the same, how far from the flagpole is Tamara standing?

21. AN APPLICATION: URBAN SPRAWL

As American cities expanded during the Twentieth Century, there were often few controls on how the land was divided. In the town of Dry Creek, one particular tract of land had the shape and dimensions shown in the figure at right. The developer planned to build five homes per acre. Determine the number of homes that can be built on this tract of land. Show all dissections and subproblems.



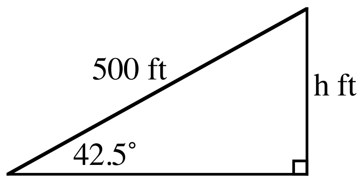
One mile is 5,280 feet and one acre contains 43,560 sq. ft

Answers

1. $h = 15 \sin 38^\circ \approx 9.235$
2. $h = 8 \sin 26^\circ \approx 3.507$
3. $x = 23 \cos 49^\circ \approx 15.089$
4. $x = 37 \cos 41^\circ \approx 27.924$
5. $y = 38 \tan 15^\circ \approx 10.182$
6. $y = 43 \tan 55^\circ \approx 61.4104$
7. $z = \frac{15}{\sin 38^\circ} \approx 24.364$
8. $z = \frac{18}{\sin 52^\circ} \approx 22.8423$
9. $w = \frac{23}{\cos 38^\circ} \approx 29.1874$
10. $w = \frac{15}{\cos 38^\circ} \approx 19.0353$
11. $x = \frac{38}{\tan 15^\circ} \approx 141.818$
12. $x = \frac{91}{\tan 29^\circ} \approx 164.168$
13. $x = \tan^{-1} \frac{5}{7} \approx 35.5377^\circ$
14. $u = \tan^{-1} \frac{7}{9} \approx 37.875^\circ$
15. $y = \tan^{-1} \frac{12}{18} \approx 33.690^\circ$

16. $v = \tan^{-1}\dfrac{78}{88} \approx 41.5526^\circ$

17.



$\sin 42.5 = \frac{h}{500}$

$h = 500 \sin 42.5^\circ \approx 337.795$ ft

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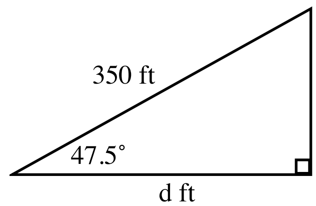

$\cos 47.5^\circ = \frac{d}{350}$

$d = 350 \cos 47.5^\circ \approx 236.46$ ft

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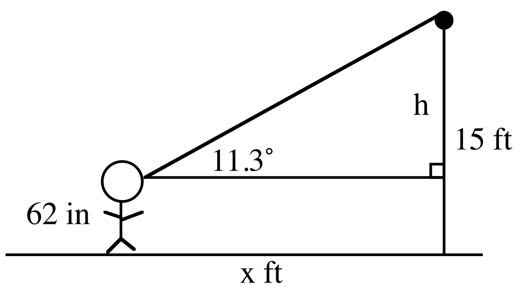
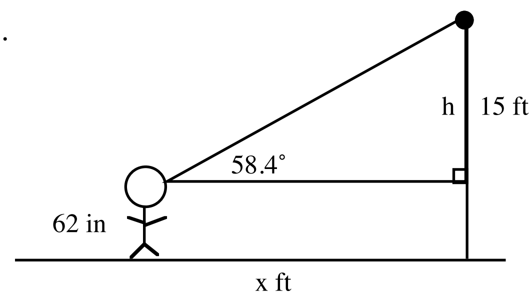


15 feet = 180 inches, $180'' - 62'' = 118'' = h$

$x \approx 590.5$ inches or 49.2 ft.

20.



$h = 118''$, $\tan 58.4^\circ = \frac{118''}{x}$,

$x \tan 58.4 = 118''$, $x = \frac{118''}{\tan 58.4^\circ}$

$x \approx 72.59$ inches or 6.05 ft.

21. (1.3 mi.)(5,280 ft./mi.) = 6,864 ft., A(45°-45°-90° Δ) = 0.5(6864)(6864) = 23,557,248 sq.'; A(sm. 30°-60°-90° Δ): hyp. = $1.3\sqrt{2}$, so h = $0.65\sqrt{2}$, b = $0.65\sqrt{6}$, A = 0.5(8406.65)(4853.58) = 20,401,175 sq.'; A(lg. 30°-60°-90° Δ): base = 1 mi., h = $\sqrt{3}$ mi., A = 0.5(5280)(9145.23) = 24,143,403; Total area = 68,101,826, divide by 43,560 = 1563.4028 acres, mult. by 5 houses per acre, 7817.014 houses, so 7817 houses.