SOLVING TRIG EQUATIONS WITH RADIANS

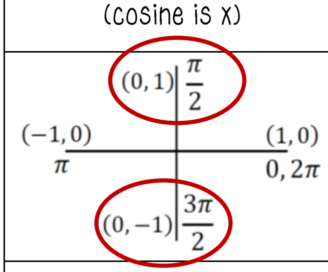
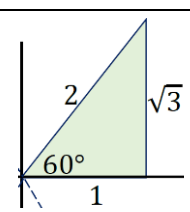
Solving *Non-Quadratic* Trigonometric Equations

1. Get the trig function alone.
2. Where are the solutions?
   a. What quadrants?
   b. OR…quadrantal angles?
3. Draw a reference triangle to determine the reference angle.
4. Use the reference angle to find the solutions in the proper quadrants.

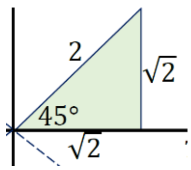
Solving *Quadratic* Trigonometric Equations

1. If necessary, use a Pythagorean Identity to rewrite the equation so trig functions are the same.
2. Set equal to zero.
3. If possible, factor.
4. Use the Zero Product Property to solve.
5. Follow the steps (above) to finish solving.

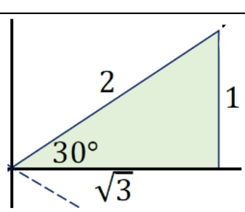
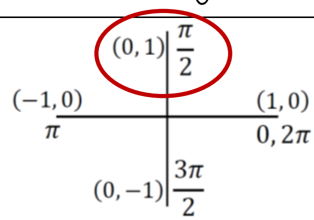
OR…Use the Square Root Property:
If $x^2 = c$, then $x = \pm c$.

EXAMPLE #1		
$2\sqrt{3}\cos\theta\sin\theta - \cos\theta = 2\cos\theta$ Subtract $2\cos\theta$ from each side. $2\sqrt{3}\cos\theta\sin\theta - 3\cos\theta = 0$		*Set equal to zero.*
$\cos\theta\left(2\sqrt{3}\sin\theta - 3\right) = 0$		*Factoring IS possible.* Common factor of $\cos\theta$.
$\cos\theta = 0$	$2\sqrt{3}\sin\theta - 3 = 0$ $\sin\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$	*Use the Zero Product Property to solve.* Simplify square roots.
Quadrantal Angle (cosine is x)	Sine is positive in quadrants 1 & 2.	*Where are the solutions?*
		*Draw a reference triangle to determine the reference angle.* Recall: sine is opposite over hypotenuse. The reference angle is 60°
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	*Use the reference angle to find the solutions in the proper quadrants.*

EXAMPLE #2

$2\sin^2\theta + 4 = 5$ $2\sin^2\theta - 1 = 0$	*Set equal to zero.*
$2\sin^2\theta = 1 \rightarrow \sin^2\theta = \frac{1}{2} \rightarrow \sqrt{\sin^2\theta} = \pm\sqrt{\frac{1}{2}}$ $\rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}} \rightarrow \sin\theta = \pm\frac{\sqrt{2}}{2}$	*Factoring isn't possible.* Since there is no "linear" term, use the Square Root Property. Simplify square roots.
Sine is positive in quadrants 1 & 2. Sine is negative in quadrants 3 & 4.	*Where are the solutions?*
	*Draw a reference triangle to determine the reference angle.* Recall: sine is opposite over hypotenuse. The reference angle is 45°
$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	*Use the reference angle to find the solutions in the proper quadrants.*

EXAMPLE #3

$\sin^2\theta + 2 - \cos^2\theta = 3\sin\theta$		
$\sin^2\theta + 2 - \cos^2\theta = 3\sin\theta$ $\sin^2\theta + 2 - (1 - \sin^2\theta) = 3\sin\theta$		*Use a Pythagorean Identity to rewrite the equation so trig functions are the same.* $\cos^2\theta = 1 - \sin^2\theta$
$\sin^2\theta + 2 - 1 + \sin^2\theta = 3\sin\theta$ $2\sin^2\theta - 3\sin\theta + 1 = 0$		Simplify/Combine Like Terms Set equal to zero.
$2x^2 - 3x + 1 = 0$ $(2x - 1)(x - 1) = 0$ $(2\sin\theta - 1)(\sin\theta - 1) = 0$		*Factoring IS possible.* Rewrite algebraically, factor, and then put $\sin\theta$ back in for x.
$2\sin\theta - 1 = 0$ $\sin\theta = \frac{1}{2}$	$\sin\theta - 1 = 0$ $\sin\theta = 1$	*Use the Zero Product Property to solve.*
Sine is positive in quadrants 1 & 2.	Quadrantal Angle (sine is y)	*Where are the solutions?*
		*Draw a reference triangle to determine the reference angle.* Recall: sine is opposite over hypotenuse. The reference angle is 30°
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	$\theta = \frac{\pi}{2}$	*Use the reference angle to find the solutions in the proper quadrants.*