

Simplifying Square Roots and Cube Roots

Square Roots	Cube Roots
<p>We know: $\sqrt{4} = 2$, since $2^2 = 4$ and $\sqrt{9} = 3$, since $3^2 = 9$.</p> <p>We also know that $\sqrt{8}$ won't come out to a whole number, but we can simplify it using $\sqrt{4}$ (because it comes out to a whole number) in the following way:</p> $\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2} \\ &= \sqrt{4} \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$ <p>We use this same principle (with other perfect square numbers) in order to simplify other square roots. Here are some other examples:</p>	<p>We know: $\sqrt[3]{8} = 2$, since $2^3 = 8$ and $\sqrt[3]{27} = 3$, since $3^3 = 27$.</p> <p>We also know that $\sqrt[3]{16}$ won't come out to a whole number, but we can simplify it using $\sqrt[3]{8}$ (because it comes out to a whole number) in the following way:</p> $\begin{aligned}\sqrt[3]{16} &= \sqrt[3]{8 \cdot 2} \\ &= \sqrt[3]{8} \sqrt[3]{2} \\ &= 2\sqrt[3]{2}\end{aligned}$ <p>We use this same principle (with other perfect cube numbers) in order to simplify other cube roots. Here are some other examples:</p>
$\begin{array}{ll}\sqrt{20} = \sqrt{4 \cdot 5} & \sqrt{18} = \sqrt{9 \cdot 2} \\ = \sqrt{4} \sqrt{5} & = \sqrt{9} \sqrt{2} \\ = 2\sqrt{5} & = 3\sqrt{2}\end{array}$	$\begin{array}{ll}\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} & \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} \\ = \sqrt[3]{8} \sqrt[3]{4} & = \sqrt[3]{27} \sqrt[3]{2} \\ = 2\sqrt[3]{4} & = 3\sqrt[3]{2}\end{array}$
$\begin{array}{ll}\sqrt{12} = \sqrt{4 \cdot 3} & \sqrt{45} = \sqrt{9 \cdot 5} \\ = \sqrt{4} \sqrt{3} & = \sqrt{9} \sqrt{5} \\ = 2\sqrt{3} & = 3\sqrt{5}\end{array}$	$\begin{array}{ll}\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} & \sqrt[3]{81} = \sqrt[3]{27 \cdot 3} \\ = \sqrt[3]{8} \sqrt[3]{5} & = \sqrt[3]{27} \sqrt[3]{3} \\ = 2\sqrt[3]{5} & = 3\sqrt[3]{3}\end{array}$
$\begin{array}{ll}\sqrt{40} = \sqrt{4 \cdot 10} & \sqrt{27} = \sqrt{9 \cdot 3} \\ = \sqrt{4} \sqrt{10} & = \sqrt{9} \sqrt{3} \\ = 2\sqrt{10} & = 3\sqrt{3}\end{array}$	$\begin{array}{ll}\sqrt[3]{48} = \sqrt[3]{8 \cdot 6} & \sqrt[3]{135} = \sqrt[3]{27 \cdot 5} \\ = \sqrt[3]{8} \sqrt[3]{6} & = \sqrt[3]{27} \sqrt[3]{5} \\ = 2\sqrt[3]{6} & = 3\sqrt[3]{5}\end{array}$
$\begin{array}{ll}\sqrt{88} = \sqrt{4 \cdot 22} & \sqrt{99} = \sqrt{9 \cdot 11} \\ = \sqrt{4} \sqrt{22} & = \sqrt{9} \sqrt{11} \\ = 2\sqrt{22} & = 3\sqrt{11}\end{array}$	$\begin{array}{ll}\sqrt[3]{88} = \sqrt[3]{8 \cdot 11} & \sqrt[3]{297} = \sqrt[3]{27 \cdot 11} \\ = \sqrt[3]{8} \sqrt[3]{11} & = \sqrt[3]{27} \sqrt[3]{11} \\ = 2\sqrt[3]{11} & = 3\sqrt[3]{11}\end{array}$
$\begin{array}{ll}\sqrt{32} = \sqrt{16 \cdot 2} & \sqrt{50} = \sqrt{25 \cdot 2} \\ = \sqrt{16} \sqrt{2} & = \sqrt{25} \sqrt{2} \\ = 4\sqrt{2} & = 5\sqrt{2}\end{array}$	$\begin{array}{ll}\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} & \sqrt[3]{128} = \sqrt[3]{64 \cdot 2} \\ = \sqrt[3]{125} \sqrt[3]{2} & = \sqrt[3]{64} \sqrt[3]{2} \\ = 5\sqrt[3]{2} & = 4\sqrt[3]{2}\end{array}$
$\begin{array}{ll}\sqrt{605} = \sqrt{121 \cdot 5} & \sqrt{432} = \sqrt{144 \cdot 3} \\ = \sqrt{121} \sqrt{5} & = \sqrt{144} \sqrt{3} \\ = 11\sqrt{5} & = 12\sqrt{3}\end{array}$	$\begin{array}{ll}\sqrt[3]{625} = \sqrt[3]{125 \cdot 5} & \sqrt[3]{768} = \sqrt[3]{64 \cdot 12} \\ = \sqrt[3]{125} \sqrt[3]{5} & = \sqrt[3]{64} \sqrt[3]{12} \\ = 5\sqrt[3]{5} & = 4\sqrt[3]{12}\end{array}$

For radicals with an index higher than three, it can be helpful to write down a short list of integers raised to the power of the index you are trying to simplify. For example when dealing with a fourth root:

$$2^4 = 16, 3^4 = 81, 4^4 = 256, 5^4 = 625, \dots$$