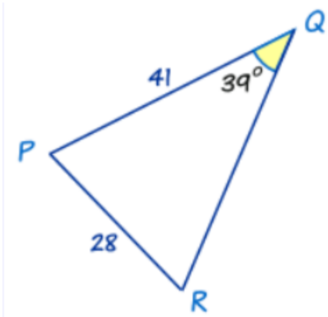
The Law of Sines – The Ambiguous Case



$$\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$$

1. Approach this problem as you would any Law of Sines problem by substituting those values you know into the Law of Sines:

$$\frac{\sin P}{p} = \frac{\sin 39}{28} = \frac{\sin R}{41}$$

2. We can solve for $\angle R$:

$$\sin R = \frac{41 \times \sin 39}{28} \approx 0.9215$$

$$m\angle R = \sin^{-1} 0.9215 \approx 67.1°$$

3. Assume that there are two triangles and find the measure of $\angle R_2$:

$$m\angle R_2 = 180 - m\angle R_1 = 180 - 67.1 = 112.9°$$

4. Now find the measure of the third angle in each triangle. (The sum of all three angles is 180°) Since the $m\angle P_2$ exists (b/c it's positive) we have two triangles.

Triangle 1

$m\angle R_1$	67.1°
$m\angle P_1$	73.9°
p_1	

Triangle 2

$m\angle R_2$	112.9°
$m\angle P_2$	28.1°
p_2	

5. Now we have to find p_1 and p_2. We need to set up and solve two proportions and use the corresponding $\angle P$.

$$\frac{\sin 73.9}{p_1} = \frac{\sin 39}{28}$$

$$p_1 = \frac{28 \times \sin 73.9}{\sin 39}$$

$$p_1 \approx 42.7$$

$$\frac{\sin 28.1}{p_2} = \frac{\sin 39}{28}$$

$$p_2 = \frac{28 \times \sin 28.1}{\sin 39}$$

$$p_2 \approx 21.0$$

6. Our solutions:

Triangle 1

$m\angle R_1$	67.1°
$m\angle P_1$	73.9°
p_1	42.7

Triangle 2

$m\angle R_2$	112.9°
$m\angle P_2$	28.1°
p_2	21.0