Name: _____

UNIT 4 ~ LINEAR EQUATIONS & INEQUALITIES

- One- & Two-Step Equations
 - Reasoning w/Equations
 - Understand solving equations as a process of reasoning & explain the reasoning
 - Solve equations in one variable
 - Creating Equations
 - Create equations in one variable and use them to solve problems
- Multi-Step Equations
 - Reasoning w/Equations
 - Understand solving equations as a process of reasoning & explain the reasoning
 - Solve equations in one variable
 - Represent and solve equations graphically
 - Creating Equations
 - Create equations in one variable and use them to solve problems
- Equations w/Variables on Both Sides
 - Reasoning w/Equations
 - Understand solving equations as a process of reasoning & explain the reasoning
 - Solve equations in one variable (include proportions)
 - Represent and solve equations graphically
 - Creating Equations
 - Create equations in one variable and use them to solve problems
- Literal Equations & Formulas
 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations
- One- & Two-Step Inequalities
 - > Solve inequalities in one variable
 - > Creating Inequalities
 - Create inequalities in one variable and use them to solve problems
- Multi-Step Inequalities
 - > Solve inequalities in one variable
 - > Creating Inequalities
 - Create inequalities in one variable and use them to solve problems
- Compound Inequalities
 - > Solve inequalities in one variable
 - Creating Inequalities
 - Create inequalities in one variable and use them to solve problems
- Linear Inequalities
 - > Graph the solution to a linear inequality in two variables as a half-plane

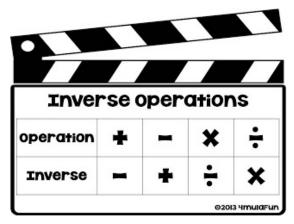
4.1 ONE- & TWO-STEP EQUATIONS

Objectives: Understand solving equations as a process of reasoning and explain the reasoning

Solve linear equations in one variable

Create equations in one variable and use them to solve problems

- Solving Equations in One Variable
 - Equivalent Equation _____
 - GOAL ~ Isolate the variable using the properties of equality and inverse operations to produce an equivalent equation
 - Inverse Operations _



2.3=6

x-7+7=x

- Properties of Equality
 - For all rational numbers a, b, and c, if a = b, then...

Properties of Equality	Symbols	Examples
Addition	If $a = b$, then $a + c = b + c$.	If $x = -4$, then $x + 4 = -4 + 4$.
Subtraction	If $a = b$, then $a - c = b - c$.	If $r + 1 = 7$, then $r + 1 - 1 = 7 - 1$.
Multiplication	If $a = b$, then $ac = bc$.	If $\frac{k}{2} = 8$, then $\frac{k}{2}(2) = 8(2)$.
Division	If $a = 2$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	If $6 = 3t$, then $\frac{6}{3} = \frac{3t}{3}$.
Reflexive	a = a	15 = 15
Symmetric	If $a = b$, then $b = a$.	If $n = 2$, then $2 = n$.
Transitive	If $a = b$ and $b = c$, then $a = c$.	If $y = 3^2$ and $3^2 = 9$, then $y = 9$.
Substitution	If $a = b$, then b can be substituted for a in any expression.	If $x = 7$, then $2x = 2(7)$.

- Algebraic Proof
 - > A proof is a logical argument that shows a conclusion is true.
 - An algebraic proof uses the properties of ______ and the ______ property.
 - To introduce a quantity to the proof, use the _____ property.
 - If a step requires simplification by combining like terms, write "_____"

examples ~ algebraic proofs

Complete the following algebraic proofs using the reasons above.

1. Given: 8 = x - 3Prove: x = 11

	Statements	Reasons
1.	8 = x - 3	1. Given
2.	x - 3 = 8	2.
3.	3 = 3	3.
4.	x - 3 + 3 = 8 + 3	4.
5.	x = 11	5.

2. Given: 2x = -10

Prove: x = -5

Sta	itements	Reasons
1.	2x = -10	1. Given
2.	$\frac{1}{2} = \frac{1}{2}$	2.
3.	$2x\left(\frac{1}{2}\right) = -10\left(\frac{1}{2}\right)$	3.
4.	x = -5	4.

3. Given:
$$\frac{x}{-3} = -12$$

Prove: $x = 36$

	Statements	Reasons
1.	$\frac{x}{-3} = -12$	1. Given
2.	-3 = -3	2.
3.	$\frac{x}{-3}(-3) = -12(-3)$	3.
4.	x = 36	4.

4. Given: 3x - 8 = 10

Prove: x = 6

S	itatements	Reasons
1.	3x - 8 = 10	1.
2.	8 = 8	2.
3.	3x - 8 + 8 = 10 + 8	3.
4.	3x = 18	4.
5.	$\frac{1}{3} = \frac{1}{3}$	5.
6.	$3x\left(\frac{1}{3}\right) = 18\left(\frac{1}{3}\right)$	6.
7.	x = 6	7.

examples ~ solving equations

Solve the following equations. Check your solution using the substitution property.

5. y - (-3) = 6 6. -8 = 6 - n

7.
$$\frac{3}{4}x = 6$$
 8. $\frac{-a}{5} = 3$

9.
$$6 - 5m = -29$$
 10. $-12 = 6 + \frac{2}{3}x$

example ~ applications & modeling

- 11. Cassidy went to the movies with some of her friends. The tickets cost \$6.50 each, and they spent \$17.50 on snacks. The total amount paid was \$63. Set up and solve an equation to determine how many tickets were purchased.
 - a. Define your variables:
 - b. Create an equation:
 - c. Solve:
 - d. Interpret:

4.2 MULTI-STEP EQUATIONS

Objectives: Understand solving equations as a process of reasoning and explain the reasoning

Solve linear equations in one variable

Create equations in one variable and use them to solve problems

Solving Multi-Step Equations

- > To solve multi-step equations, form a series of simpler equivalent equations.
 - Use the properties of equality, inverse operations, and properties of real numbers
 - Simplify by _____

Remove grouping symbols using the _____

examples ~ algebraic proofs

Complete the following algebraic proofs using the reasons from Lesson 4.1.

1. Given:
$$5 = 5m - 23 + 2m$$

Prove: m = 4

	Statements		Reasons
1.	5 =	5m - 23 + 2m	1.
2.	5m - 23 + 2m =	5	2.
3.	5m + 2m - 23 =	5	3. Commutative Property
4.	7m - 23 =	5	4.
5.	23 =	23	5.
6.	7m - 23 + 23 =	5 + 23	6.
7.	7m =	28	7.
8.	$\frac{1}{7} =$	$\frac{1}{7}$	8.
9.	$7m\left(\frac{1}{7}\right) =$	$28\left(\frac{1}{7}\right)$	9.
10.	m =	4	10.
Solve:			

2. 11x - 8 - 6x = 22

3.-2y+5+5y=14

examples ~ algebraic proofs

Complete the following algebraic proofs using the reasons from Lesson 4.1.

4.	. Given: $2(a + 1) = -6$ Prove: $a = -4$			
		Statements	Reasons	
	1.	2(a+1) = -6	1.	
	2.	2a + 2 = -6	2.	
	3.	-2 = -2	3.	
	4.	2a + 2 + (-2) = -6 + (-2)	2) 4.	
	5.	2a = -8	5.	
	6.	$\frac{1}{2} = \frac{1}{2}$	6.	
	7.	$2a\left(\frac{1}{2}\right) = -8\left(\frac{1}{2}\right)$	7.	
	8.	a = -4	8.	
6.0	h (0-			

Solve:

5.-8(2x-1) = 36

6.-24 = 5(y+3)

examples ~ applications & modeling

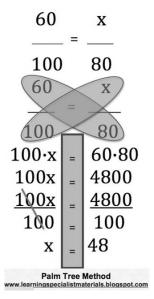
- 7. Martha takes her niece and nephew to a concert. She buys T-shirts and bumper stickers for them. The bumper stickers cost \$1 each. Martha's niece wants one shirt and 4 bumper stickers, and her nephew wants two shirts but no bumper stickers. If Martha's total is \$67, what is the cost of a T-shirt?
 - a. Define your variables:
 - b. Create an equation:
 - c. Solve:
 - d. Interpret:

- 8. Three friends go bowling. The cost per person per game is \$5.30. The cost to rent shoes is \$2.50 per person. Their total cost is \$55.20. How many games did they play?
 - a. Define your variables:
 - b. Create an equation:
 - c. Solve:
 - d. Interpret:
- Solving Proportions
 - A proportion is an equation that states that two ratios are equal.
 - Cross Product Property

If
$$\frac{a}{b} = \frac{c}{d}$$
, where $b \neq 0 \& d \neq 0$, then $ad = bc$.

examples ~ solving proportions

9.
$$\frac{-3}{4} = \frac{m}{22}$$



10.
$$\frac{a-2}{9} = \frac{2}{3}$$

EXAMPLE ~ APPLICATIONS & MODELING

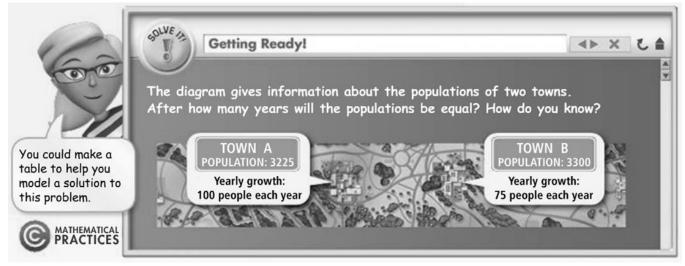
- 11. A portable media player has 2 gigabytes of storage and can hold about 500 songs. A similar but larger media player has 8 gigabytes of storage. About how many songs can the larger media player hold?
 - a. Define your variables:
 - b. Create an equation:
 - c. Solve:
 - d. Interpret:

4.3 EQUATIONS WITH VARIABLES ON BOTH SIDES

Objectives: Understand solving equations as a process of reasoning and explain the reasoning

Solve linear equations in one variable

Create equations in one variable and use them to solve problems



- a) Write an equation in slope-intercept form that models the population of Town A.
- b) Write an equation in slope-intercept form that models the population of Town B.
- c) Complete the tables below:

Т	Town A		
Year	Population		
1			
ณ			
3			
4			
5			

Town B		
Year Population		
1		
2		
3		
4		
5		

d) After how many years will the population be equal? How do you know?

- Solving Equations with Variables on Both Sides
 - To solve equations with variables on both sides, you can use the properties of equality and inverse operations to write a series of simpler equivalent equations.

examples ~ algebraic proofs

1. Given: 5x + 2 = 2x + 14

Complete the following algebraic proofs using the reasons above.

1.		x = 4		
		Statements		Reasons
	1.	5x + 2 =	2x + 14	1.
	2.	-2x =	-2x	2.
	3.	5x + 2 + (-2x) =	2x + 14 + (-2x)	3.
	4.	5x + (-2x) + 2 =	2x + (-2x) + 14	4. Commutative Property
	5.	3x + 2 =	14	5.
	6.	-2 =	-2	6.
	7.	3x + 2 + (-2) =	14 + (-2)	7.
	8.	3x =	12	8.
	9.	$\frac{1}{3} =$	$\frac{1}{3}$	9.
	10.	$3x\left(\frac{1}{3}\right) =$	$12\left(\frac{1}{3}\right)$	10.
	11.	<i>x</i> =	4	11.

Solving Equations Summary

Use the ______ to remove any grouping symbols.

Image: A state of the equation.

- Use the properties of ______ to get the variable terms on one side of the equation and the constants on the other.
- > Use the properties of ______ to solve for the variable.
- > Check your solution in the original equation.

EXAMPLES ~ **ALGEBRAIC PROOFS**

Complete the following algebraic proofs using the reasons above.

2.		en: $3x - 10 = 5(x - 4)$ ve: $x = 5$		
		Statements		Reasons
	1.	3x - 10 =	5(x-4)	1.
	2.	3x - 10 =	5x - 20	2.
	3.	-5x =	-5x	3.
	4.	3x - 10 + (-5x) =	5x - 20 + (-5x)	4.
	5.	3x + (-5x) - 10 =	5x + (-5x) - 20	5. Commutative Property
	6.	-2x - 10 =	-20	6.
	7.	10 =	10	7.
	8.	-2x - 10 + 10 =	-20 + 10	8.
	9.	-2x =	-10	9.
	10.	$-\frac{1}{2} =$	$-\frac{1}{2}$	10.
	11.	$-2x\left(-\frac{1}{2}\right) =$	$-10\left(-\frac{1}{2}\right)$	11.
	12.	x =	5	12.

Solve:

3. 7y + 5 = 2y + 10

4. 3n - 15 = 5n + 3 - 4n

5. 3x - 3 = 3(7 - x)6. 5(m + 4) = 7(m - 2) 7. 3(7 + 2a) = 30 + 7(a - 1)

EXAMPLE ~ APPLICATIONS & MODELING

- 8. A skier is trying to decide whether or not to buy a season ski pass. A daily pass costs \$67. A season ski pass costs \$350. The skier would have to rent skies with either pass for \$25 per day. How many days would the skier have to go skiing in order to make the season pass less expensive than the daily pass?
 - a. Define your variables:
 - b. Create two equations:
 - c. Solve:
 - d. Interpret:

4.4 EQUATIONS WITH FRACTIONS & DECIMALS

Objectives: Understand solving equations as a process of reasoning and explain the reasoning

Solve linear equations in one variable

Create equations in one variable and use them to solve problems

ACCESSING PRIOR KNOWLEDGE

Simplify:

1.
$$3 \times \frac{1}{3}x$$
 2. $2\left(\frac{1}{2}y + \frac{3}{2}\right)$ 3. $10(1.2u + 0.5)$

- Clearing an Equation of Fractions
 - Multiply both sides of the equation by the ______
 of all the fractions in the equation
 - What is the multiplier that will eliminate the equation of fractions?

$$\frac{3}{4}x - 7 = 8 + \frac{2}{3}x$$

EXAMPLES ~ SOLVING EQUATIONS WITH FRACTIONS

1.
$$\frac{1}{3}x + \frac{1}{6} = \frac{3}{2}$$
 2. $\frac{1}{2} + 4a = 3a - \frac{5}{2}$

3.
$$\frac{7}{8}m + \frac{3}{4} = \frac{1}{2}m + \frac{3}{2}$$

- Clearing an Equation of Decimals
 - Multiply both sides of the equation by the appropriate _____
 - What is the multiplier that will eliminate the equation of decimals? 0.21x + 4.52 = -0.73 - 0.84x

examples ~ solving equations with decimals

4. 16.3 - 7.2y = -8.18 5. 26.45 = 4.2x + 1.25

6. 41.68 = 4.7 - 8.6y

4.5 LITERAL EQUATIONS & FORMULAS

- Objectives: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations
- Literal Equations
 - A <u>literal equation</u> is an equation that involves two or more variables.

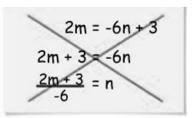
examples ~ rewriting a literal equation

- 1. You are ordering pizzas and sandwiches. You have a budget of \$80.
 - a. Write an equation in standard form that models this situation, where x is the number of pizzas and y is the number of sandwiches.



- b. How many sandwiches can you buy is you buy 3 pizzas?
- c. Six pizzas?
- 2. Solve each equation for the given variable.
 - a. -2x + 5y = 12 for y
 - b. a 2b = -10 for *b*
 - c. mx + 2n = p for x

3. Describe and correct the error made in solving the literal equation for n.



Formulas

 \succ A <u>formula</u> is an equation that states a relationship among quantities.

Formula Name	Formula	Definitions of Variables
Perimeter of a rectangle	$P=2\ell+2w$	$P =$ perimeter, $\ell =$ length, $w =$ width
Circumference of a circle	$C = 2\pi r$	C = circumference, r = radius
Area of a rectangle	$A = \ell w$	$A = \text{area}, \ell = \text{length}, w = \text{width}$
Area of a triangle	$A = \frac{1}{2}bh$	A = area, b = base, h = height
Area of a circle	$A = \pi r^2$	A = area, r = radius
Distance traveled	d = rt	d = distance, r = rate, t = time
Temperature	$C=\frac{5}{9}(F-32)$	C = degrees Celsius, $F =$ degrees Fahrenheit

examples ~ rewriting a formula

4. What is the radius of a circle with circumference of 64 feet? Round to the nearest tenth. Use 3.14 for π .

5. What is the height of a triangle that has an area of 24 square inches and a base of length 8 inches?

6. The monarch butterfly is the only butterfly that migrates annually north and south. The distance that a particular group of monarch butterflies travels is shown. It takes a typical butterfly about 120 days to travel one way. What is the average rate at which a butterfly travels in miles per day? (Round to the nearest mile per day.)



7. Jonah is planting a rectangular garden. The perimeter of the garden is 120 yards and the width if 20 yards. What is the length of the garden?

examples ~ rewriting linear equations

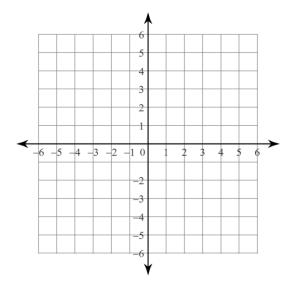
Rewrite each linear equation, given in standard form Ax + By = C, in slope-intercept form: y = mx + b, and then graph.

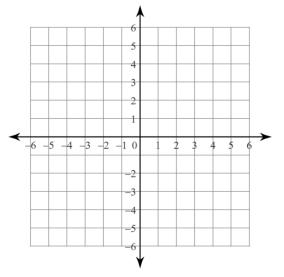
8. 5x - 3y = -3

Slope-intercept form:

9. 4x - 3y = -6

Slope-intercept form:





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4.6 ONE- & TWO-STEP INEQUALITIES

Objectives: Solve an inequality using the addition and multiplication principles and then graph the solution set

Create inequalities in one variable and use them to solve problems

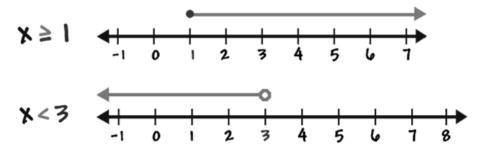
- Inequalities
 - > An inequality is any sentence containing $<, >, \le, \text{ or } \ge$.
 - Solution ~ any replacement for the variables that makes an inequality true
 - Solution Set ~ the set of all solutions

EXAMPLES ~ SOLUTIONS OF INEQUALITIES

Determine whether each number is a solution of the given inequality.

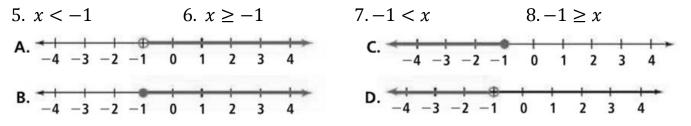
1. $3y - 8 > 22$	a. 2	<i>b</i> . 0	<i>c</i> . 5
2. $8m - 6 \le 10$	<i>a</i> . 2	<i>b</i> . 3	<i>c</i> .−1
3. $4x + 2 < -6$	<i>a</i> . 0	b2	<i>c</i> . 1
4. $n(n-3) < 54$	a10	<i>b</i> . 0	c. 9

- Graphing Inequalities:
 - > Open circle: < or >
 - \blacktriangleright Closed circle: \leq or \geq



examples ~ graphs of inequalities

Match each inequality with its graph.





Schultz says:

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SOLVE INEQUALITIES AS YOU
WOULD EQUATIONS...
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...BUT BE CAREFUL WHEN MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER.

$$2 \times -5 \ge 7$$

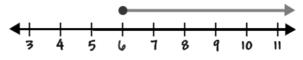
+5+5 ditch the 5
$$2 \times \ge 12$$

$$\frac{2 \times}{2} \ge \frac{12}{2}$$
 ditch the 2
$$\times \ge 6$$

OK, so what does this answer mean?

(It's super important in math to understand what your answers mean!)

We can graph it on a number line:



So, in our original problem, $2x - 5 \ge 7$, x can be 6... or X can be a number bigger than 6.



Solve

$$-3 \times \leq 6$$

 $-3 \times \leq 6$ ditch the -3
 $-3 -3$
 $\times \leq -2$

It looks ok ... But, is it?

This means that X can be -2 or any other number less than -2.

Let's check!

$$-3 \times \leq 6$$

$$\chi = -2 \longrightarrow -3(-2) \leq 6$$

$$6 \leq 6 \quad \text{Yep-that works.}$$

$$\chi = -4 \longrightarrow -3(-4) \leq 6$$

$$12 \leq 6 \quad \text{NO WAY. DUDE!}$$

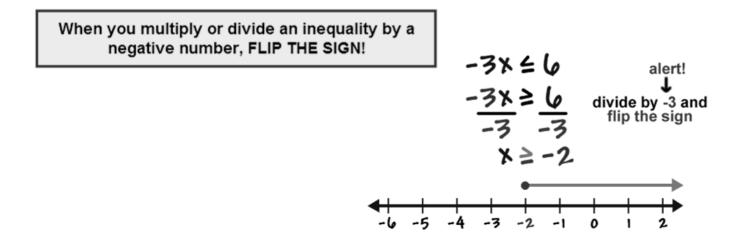
Schultz says: SOLVE INEQUALITIES AS YOU WOULD EQUATIONS...

...BUT BE CAREFUL WHEN MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER.

It didn't work. Wazzup with that?

Here's the freaky thing:

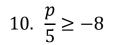
When you divide (or multiply) by a negative number, you mess up the inequality sign!



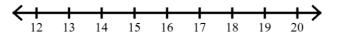
examples ~ solving inequalities

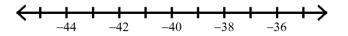
Solve each inequality and graph the solution set.

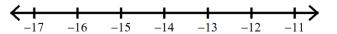
9.-2 < x - 18



11.8n - 4 < -116

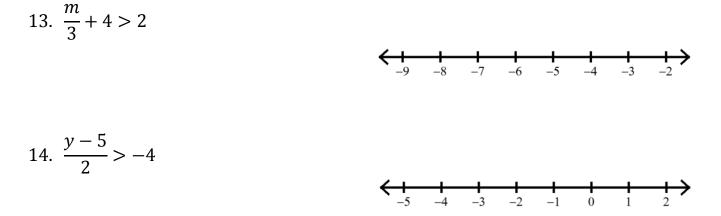






12. $6 - 6x \le -54$





examples ~ applications of inequalities

15. Write an inequality that describes the situation.





- 16. The hard drive on your computer has a capacity of 120 gigabytes (GB). You have used 85 GB. You want to save some home videos to your hard drive. What are the possible sizes of the home video collection you can save
 - a. Define your variables:
 - b. Create an inequality:
 - c. Solve:
 - d. Interpret:
- 17. You walk dogs in your neighborhood after school. You earn \$4.50 per dog. How many dogs do you need to walk to earn at least \$75?
 - a. Define your variables:
 - b. Create an inequality:
 - c. Solve:
 - d. Interpret:

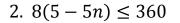
4.7 MULTI-STEP INEQUALITIES

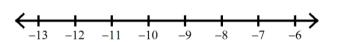
Objectives: Solve an inequality using the addition and multiplication principles and then graph the solution set Create inequalities in one variable and use them to solve problems

examples ~ solving inequalities

Solve each inequality and graph the solution set.

1. $-2x + 8x \ge 8x + 12$

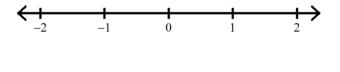




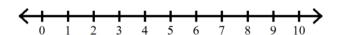
3. $26 + 8m \ge 5(1 + 3m)$



4.-5+7n > 7-6(6n+2)

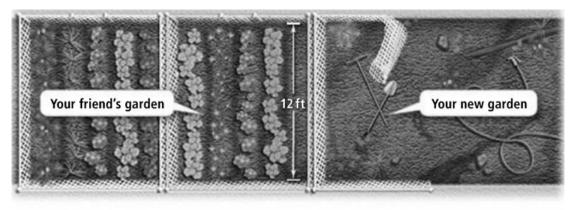


5.
$$14 < 8(x+8) - 5(8+2x)$$



examples ~ applications & modeling

6. In a community garden, you want to fence in a vegetable garden that is adjacent to your friend's garden. You have at most 42 feet of fence. What are the possible lengths of your garden?



Relate Since the fence will surround the garden, you can use the perimeter formula $P = 2\ell + 2w$.

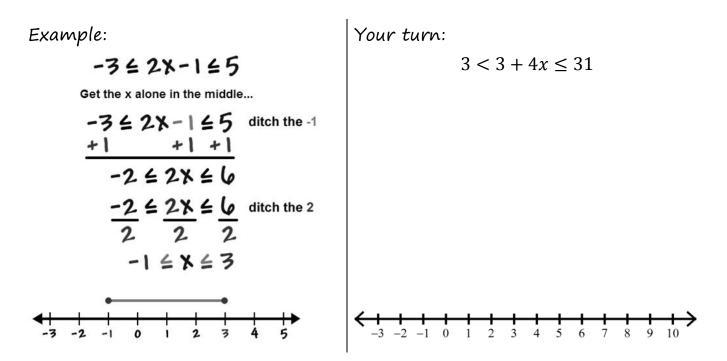


- 7. On a trip from Buffalo, New York to St. Augustine, Florida, a family wants to travel at least 250 miles in the first 5 hours of driving. What should their average speed be in order to meet this goal?
 - a. Define your variables:
 - b. Create an inequality:
 - c. Solve:
 - d. Interpret:
- 8. A sales associate in a shoe store earns \$325 per week, plus a commission equal to 4% of her sales. This week her goal is to earn at least \$475. At least how many dollars' worth of shoes must she sell in order to reach her goal?
 - a. Define your variables:
 - b. Create an inequality:
 - c. Solve:
 - d. Interpret:

4.8 COMPOUND INEQUALITIES

Objectives: Solve compound inequalities and then graph the solution set Create inequalities in one variable and use them to solve problems

- Compound Inequality
 - Consist of two or more inequalities joined by the word "and" or the word "or"
 - > Conjunction
 - When two or more sentences are joined by the word "and"
 - -2 < x and $x < 1 \leftrightarrow -2 < x < 1$
 - The solution set of a conjunction is the intersection of the solution sets.



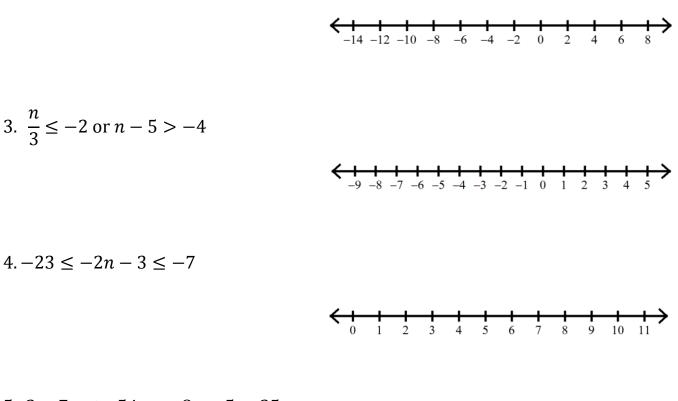
d Compound Inequality

- Consist of two or more inequalities joined by the word "and" or the word "or"
- ➢ Disjunction
 - When two or more sentences are joined by the word "or"
 - -2 < *x* or *x* < 1
 - The solution set of a disjunction is the union of the individual solution sets.

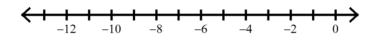
examples ~ solving inequalities

Solve each inequality and graph the solution set.

 $2 - 7x \le -28$ or x + 3 < -7



5.
$$2 + 7x \le -54$$
 or $-8x - 5 < 35$

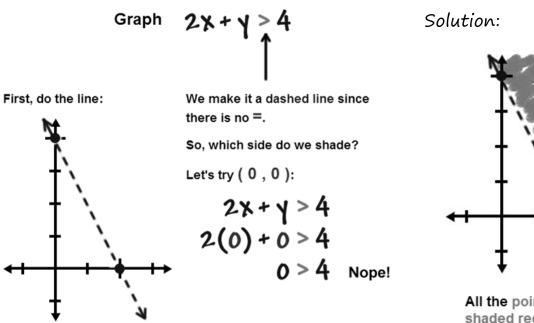


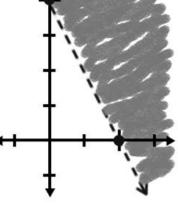
examples ~ applications & modeling

- 6. To earn a B in your Algebra course, you must achieve an unrounded test average between 84 and 86, inclusive. You scored 86, 85, and 80 on the first three tests of the grading period. What possible scores can you earn on the fourth and final test to earn a B in the course?
 - a. Define your variables:
 - b. Create an inequality:
 - c. Solve:
 - d. Interpret:

4.9 LINEAR INEQUALITIES

Objective: Graph the solution to a linear inequality in two variables as a half-plane





All the points in the shaded region work in

($0\,$, $\,0\,$) doesn't work! That means nothing on that side will work ... So, all the good points must be on the other side!

2x+y>4

examples ~ identifying solutions of a linear inequality

Determine whether the ordered pair is a solution of the linear inequality.

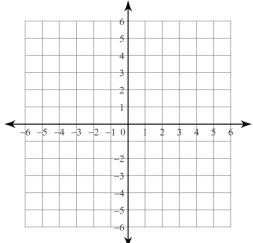
1. $y \le \frac{2}{3}s + 4$; (3,6) 2. y < 2x + 5; (-1,4)

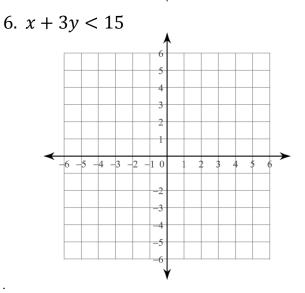
3. To graph the inequality $y < \frac{3}{2}x + 3$, do you shade above or below the boundary line? Explain.

examples ~ graphing linear inequalities

Graph the solution of each linear equality.

4. y < 5x - 5





EXAMPLES ~ APPLICATIONS & MODELING

- 8. For a party, you can spend no more than \$12 on nuts. Peanuts (x)cost
 \$2/pound and cashews (y) cost
 \$4/pound.
 - a. Write an inequality in standard form to model this situation.
 - b. What are three possible combinations of peanuts and cashews you can buy?

