How to use the definitions of the trigonometric functions to prove an identity:

Definition of Trigonometric Functions

If θ is an angle in standard position and if (x, y) is any point on the terminal side of θ such that:

$r = \sqrt{x^2 + y^2}$, *then…*

Sine	$\sin\theta = \frac{y}{r}$	**Cosecant**	$\csc\theta = \frac{r}{y}$
Cosine	$\cos\theta = \frac{x}{r}$	**Secant**	$\sec\theta = \frac{r}{x}$
Tangent	$\tan\theta = \frac{y}{x}$	**Cotangent**	$\cot\theta = \frac{x}{y}$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Given

$$\cot\theta = \frac{\frac{x}{r}}{\frac{y}{r}} \rightarrow \frac{x}{r} \div \frac{y}{r}$$

Substitute in the definitions of sinθ & cosθ.

A fraction indicates division, so I rewrote the expression to show this more clearly.

$$\cot\theta = \frac{x}{r} \times \frac{r}{y} = \frac{xr}{ry}$$

When you divide fractions, you multiply the first fraction by the reciprocal of the second fraction.

$$\cot\theta = \frac{x}{y}$$

After multiplying and simplifying, I have the definition of cotθ.

$$1 = \sin^2\alpha + \cos^2\alpha$$

Given.

$$1 = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

Substitute in the definitions of sinθ & cosθ.

$$1 = \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

Simplify.

$$1 = \frac{y^2 + x^2}{r^2}$$

Add the fractions.

$$1 = \frac{r^2}{r^2}$$

We know that $r^2 = x^2 + y^2$ so we make this substitution.

$$1 = 1$$