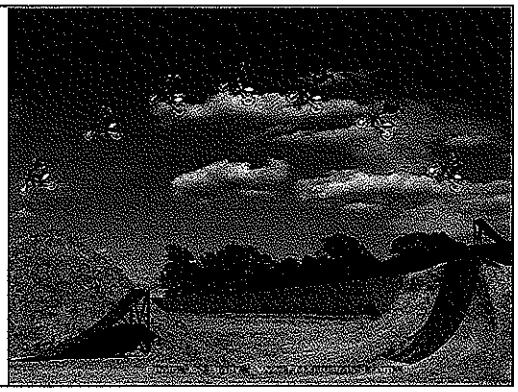
Algebra 1 **Problem-Solving Workshop**

Vertical Motion Models

A *projectile* is an object that is propelled into the air but has no power to keep itself in the air. A thrown ball is a projectile, but an airplane is not. The height h (in feet) of a projectile can be modeled by

$h = -16t^2 + vt + s$

where t is the time (in seconds), v is the initial vertical velocity (in feet per second), and s is the initial height (in feet).

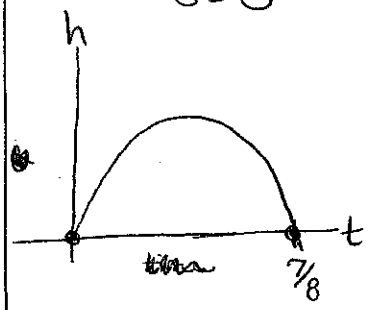


For each problem scenario, write a vertical motion equation, sketch a graph of the projectile's motion, and solve appropriately.

1. A startled armadillo jumps straight into the air with an initial vertical velocity of 14 feet per second. After how many seconds does it land on the ground?

$V = 14$
$S = 0$

→ Solve for t when h=0



$h = -16t^2 + 14t + 0$

$0 = -16t^2 + 14t$

$0 = -2t(8t - 7)$

$-2t = 0 \quad 8t - 7 = 0$

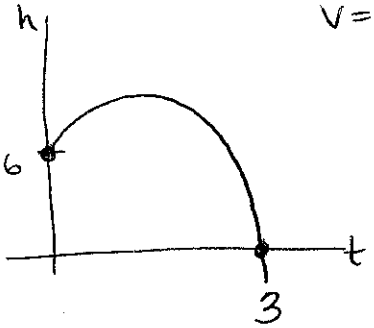
$t = 0 \quad 8t = 7$

$t = \frac{7}{8}$

$\frac{7}{8}$ second

2. An athlete throws a discus from an initial height of 6 feet and with an initial vertical velocity of 46 feet per second. Write an equation that gives the height in feet of the discus as a function of time (in seconds) since it left the athlete's hand. After how many seconds does the discus hit the ground? → solve for t when h=0

$S = 6$
$V = 46$



$h = -16t^2 + 46t + 6$

$0 = -16t^2 + 46t + 6$

$0 = -2(8t^2 - 23t - 3)$

$ac = -24$
$b = -23$
$1, -24$

$0 = (8t^2 + t)(24t - 3)$

$0 = t(8t + 1) - 3(8t + 1)$

$0 = (t - 3)(8t + 1)$

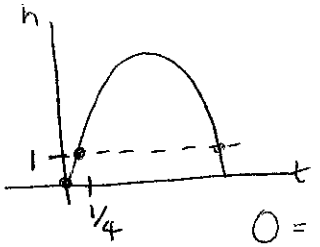
$t - 3 = 0 \quad 8t + 1 = 0$

$t = 3 \quad 8t = -1$

$t = -\frac{1}{8}$

3 seconds

3. A grasshopper jumps straight up from the ground with an initial vertical velocity of 8 feet per second. After how many seconds will the grasshopper be 1 foot off of the ground?



$s=0$
$v=8$

$h=-16t^2+8t+0$

$1=-16t^2+8t$

$0=-16t^2+8t-1$

$ac=16$, $b=-8$, $-4,-4$

$0=-(16t^2-8t+1)$

$0=(16t^2-4t)(-4t+1)$

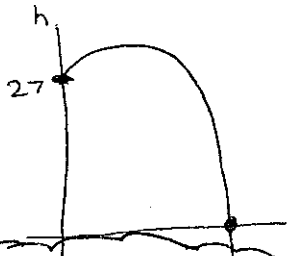
$0=4t(4t-1)-1(4t-1)$

$0=(4t-1)(4t-1)$

$4t-1=0$

$t=1/4$ sec

4. A squirrel is 27 feet up in a tree and tosses a nut out of the tree with an initial vertical velocity of 6 feet per second. The squirrel climbs down the tree in 2 seconds. Does it reach the ground before the nut?



$s=27$
$v=6$

$h=-16t^2+6t+27$

$0=-16t^2+6t+27$

$ac=-432$, $b=-6$, $-24,18$

$0=-(16t^2-6t-27)$

$0=(16t^2-24t)+(18t-27)$

$0=8t(2t-3)+9(2t-3)$

$0=(8t+9)(2t-3)$

$8t+9=0$

$8t=-9$

$t=-9/8$

$2t-3=0$

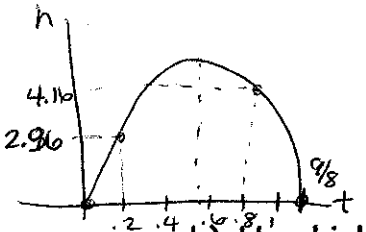
$t=3/2$

The nut reaches the ground first

5. A kangaroo can jump with an initial vertical velocity of 18 feet per second.

$v=18$
$s=0$

a) When will the kangaroo land on the ground?



$h=-16t^2+18t+0$

$0=-16t^2+18t$

$0=-2t(8t-9)$

$-2t=0$

$t=0$

$8t-9=0$

$t=9/8$ second

b) How high is the kangaroo after 0.2 seconds?

$t=0.2$

$h=-16(0.2)^2+18(0.2)+0$

$h=2.96$ feet

c) How high is the kangaroo after 0.8 seconds?

$t=0.8$

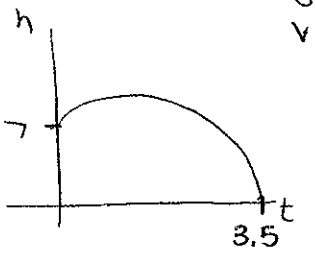
$h=-16(0.8)^2+18(0.8)+0$

$h=4.16$

d) When does the kangaroo reach its maximum height? Approximately how high does it jump?

~ around .5 → .6 seconds the kangaroo will reach its maximum height, which is about 5 feet

6. A baseball player releases a baseball at a height of 7 feet with an initial velocity of 54 feet per second. How long will it take the ball to reach the ground?



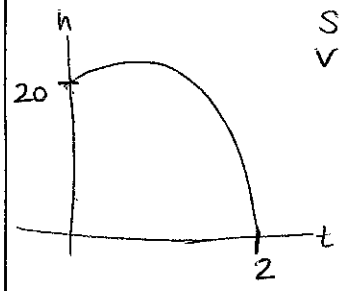
$S = 7$
$V = 54$

$h = -16t^2 + 54t + 7$
$0 = -16t^2 + 54t + 7$
$0 = -(16t^2 - 54t - 7)$
$0 = (16t^2 + 2t) - (56t - 7)$
$0 = 2t(8t+1) - 7(8t+1)$
$0 = (2t-7)(8t+1) = 0$
$t = 7/2 \qquad t = -1/8$

$ac = -112$
$b = -54$
$2, -56$

3.5 seconds

7. A miniature rocket is launched off a roof 20 feet above the ground with an initial velocity of 22 feet per second. How much time will elapse before the rocket reaches the ground?



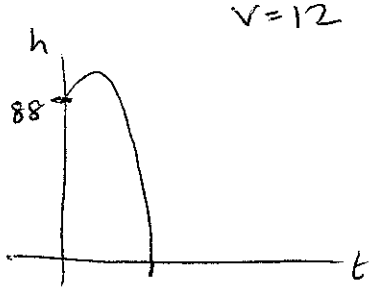
$S = 20$
$V = 22$

$h = -16t^2 + 22t + 20$
$0 = -16t^2 + 22t + 20$
$0 = -2(8t^2 - 11t - 10)$
$0 = (8t^2 - 16t) + (5t - 10)$
$0 = 8t(t-2) + 5(t-2)$
$0 = (8t+5)(t-2) = 0$
$t = -5/8 \qquad t = 2$

$ac = -80$
$b = -11$
$8, -10$
$5, -16$

2 seconds

8. A cliff diver jumps from a ledge 88 feet above the ocean with an initial upward velocity of 12 feet per second. How long will it take until the diver enters the water?



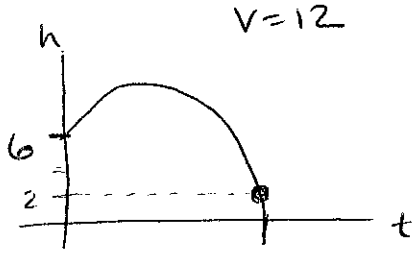
$S = 88$
$V = 12$

$h = -16t^2 + 12t + 88$
$0 = -16t^2 + 12t + 88$
$0 = -4(4t^2 - 3t - 22)$
$0 = (4t^2 + 8t) - (11t - 22)$
$0 = 4t(t+2) - 11(t+2)$
$0 = (4t-11)(t+2) = 0$
$t = 11/4 \qquad t = -2$

$ac = -88$
$b = -3$
$8, -11$

2.75 seconds

9. You throw a football from a height of 6 feet into the air with an initial vertical velocity of 12 feet per second. The football is caught at a height of 2 feet. After how many seconds is the football caught?



$S = 6$
$V = 12$

$h = -16t^2 + 12t + 6$
$2 = -16t^2 + 12t + 6$
$16t^2 - 12t - 4 = 0$
$-4(4t^2 - 3t - 1) = 0$
$(4t^2 - 4t) + (t - 1) = 0$
$4t(t-1) + 1(t-1) = 0$
$0 = (4t+1)(t-1) = 0$
$t = -1/4 \qquad t = 1$

$ac = -4$
$b = -3$
$1, -4$

1 second

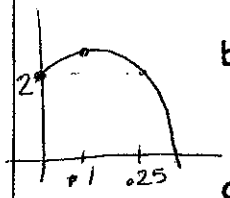
10. You hit a badminton birdie upward with a racket from a height of 2 feet with an initial velocity of 4 feet per second.

$s = 2$, $v = 4$

$h = -16t^2 + 4t + 2$

a) How high is the birdie at 0.1 seconds?

$h = -16(0.1)^2 + 4(0.1) + 2 =$ 2.24 feet



b) How high is the birdie at 0.25 seconds?

$h = -16(0.25)^2 + 4(0.25) + 2 =$ 2 feet

c) How long will it take the birdie to reach the ground? 0.5 seconds

$0 = -16t^2 + 4t + 2$

$0 = -2(16t^2 - 4t - 2)$

$0 = (16t^2 + 4t)(-8t - 2)$

$0 = 4t(4t+1) - 2(4t+1)$

$0 = (4t-2)(4t+1)$

$4t - 2 = 0$ $\quad$ $4t + 1 = 0$

$t = 2/4$ $\quad$ $t = -1/4$

$t = 1/2$

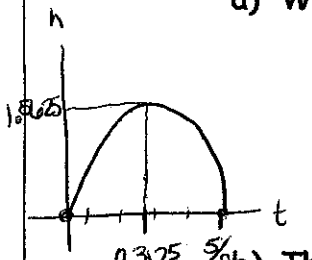
$a \cdot c = -32$, $b = -4$, $4, -8$

11. A spittlebug jumps into the air with an initial vertical velocity of 10 feet per second.

$v = 10$

$s = 0$

a) When will the spittlebug land on the ground?



$h = -16t^2 + 10t + 0$

$0 = -16t^2 + 10t$

$0 = -2t(8t - 5)$

$-2t = 0$ $\quad$ $8t - 5 = 0$

$t = 0$ $\quad$ $8t = 5$

$t = 5/8$ second

b) The spittlebug reaches its maximum height after 0.3125 second. How high can it jump?

$t = 0.3125$

$h = -16(0.3125)^2 + 10(0.3125)$

$h = 1.5625$ feet

c) How does the time in part (b) relate to the time in part (a)?

0.3125 $\quad$ $5/8 = .625$

$0.3125 = \frac{1}{2}(.625)$

d) What relationship do you think there is between these coordinates in parts (a) and (b)? Examine a sketch if the spittlebug's jump to help.

The maximum height occurs halfway between the 2 x-intercepts (when the bug starts & lands the jump)