

Write the following expression in terms of sine.

$$\frac{1}{1 + \cot \theta} + \frac{1}{1 - \cot \theta}$$

$$(1 + \cot \theta)(1 - \cot \theta)$$

Find a common denominator

$$\frac{1(1 - \cot \theta)}{(1 + \cot \theta)(1 - \cot \theta)} + \frac{1(1 + \cot \theta)}{(1 - \cot \theta)(1 + \cot \theta)}$$

Write each expression as an equivalent expression w/this common denominator.

$$\frac{1 - \cot \theta + 1 + \cot \theta}{(1 + \cot \theta)(1 - \cot \theta)}$$

Add the numerators; write the sum over the common denominator.

$$\frac{2}{1 - \cot^2 \theta}$$

Simplify.

$$\begin{aligned} \frac{2}{1 - \cot^2 \theta} &= \frac{2}{1 - (\csc^2 \theta - 1)} \\ &= \frac{2}{1 - \csc^2 \theta + 1} = \frac{2}{2 - \csc^2 \theta} \end{aligned}$$

Apply a Pythagorean Identity:

$$1 + \cot^2 \theta = \csc^2 \theta$$

Solve this for cotangent:

$$\cot^2 \theta = \csc^2 \theta - 1$$

Substitute & simplify.

$$\frac{2}{2 - \csc^2 \theta} = \frac{2}{2 - \frac{1}{\sin^2 \theta}}$$

We know that:

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\frac{2 \times \sin^2 \theta}{2 \times \sin^2 \theta - \frac{1}{\sin^2 \theta} \times \sin^2 \theta}$$

Multiply all 3 terms by $\sin^2 \theta$

$$\frac{2\sin^2 \theta}{2\sin^2 \theta - 1}$$

Write the following expression in terms of cosine.

$$\sin^4 \alpha - 2\sin^2 \alpha + 1$$

$$(\sin^2 \alpha - 1)(\sin^2 \alpha - 1)$$

Factor. (Think in terms of x .)

$$x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1)$$

$$(1 - \cos^2 \alpha - 1)(1 - \cos^2 \alpha - 1)$$

$$(-\cos^2 \alpha)(-\cos^2 \alpha)$$

$$\cos^4 \alpha$$

Apply a Pythagorean Identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

Solve for sine:

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Substitute & simplify.

Write the following expression in terms of tangent.

$$\frac{\sec \theta}{\sin \theta}$$

$$\frac{\sec \theta \times \sec \theta}{\sin \theta \times \sec \theta} = \frac{\sec^2 \theta}{\sin \theta \sec \theta}$$

Multiply the numerator and denominator by $\sec \theta$ (we're technically multiplying by 1)

$$\frac{\sec^2 \theta}{\sin \theta \sec \theta} = \frac{\tan^2 \theta + 1}{\sin \theta \times \frac{1}{\cos \theta}} = \frac{\tan^2 \theta + 1}{\frac{\sin \theta}{\cos \theta}}$$

Apply a Pythagorean Identity:

$$\tan^2 \theta + 1 = \sec^2 \theta$$

Rewrite $\sec \theta$ in terms of $\cos \theta$

Substitute & simplify.

$$\frac{\tan^2 \theta + 1}{\tan \theta}$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

Substitute.