has three sides but has three angles as well. Can you name the angle at the top of the triangle shown on the preceding page in three ways?

The triangle is the ***union*** (∪) of three segments.

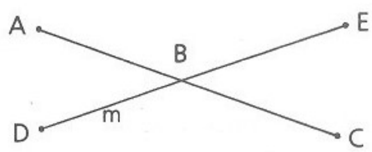
$$\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{AC}$$

The ***intersection*** (∩) of any two sides is a ***vertex*** of the triangle.

$$\overline{AB} \cap \overline{BC} = B$$

Part Two: Sample Problems

Problem 1



a *How many lines are shown? (Imagine that there are arrows in the diagram.)*

b *Name these lines.*

c *Where do* $\overleftrightarrow{AC}$ *and* $\overleftrightarrow{DE}$ *intersect?*

d *Where does* $\overrightarrow{AC}$ *intersect* $\overleftrightarrow{BC}$*?* ($\overrightarrow{AC} \cap \overleftrightarrow{BC} =$ __?__)

e *What is the union of* $\overrightarrow{BA}$ *and* $\overrightarrow{BD}$*?* ($\overrightarrow{BA} \cup \overrightarrow{BD} =$ __?__)

Answers

a 2

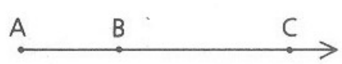
b Line m, $\overleftrightarrow{DB}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{BE}$, $\overleftrightarrow{EB}$, or $\overleftrightarrow{ED}$; $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$, or $\overleftrightarrow{CB}$

c B

d $\overrightarrow{AC}$ (Remember sets? If P and Q are two sets of points, then $P \cap Q$ = {all points in P *and* in Q}.)

e ∠ABD ($P \cup Q$ = {all points in P *or* in Q or in both}.)

Problem 2



a *Name the ray that has endpoint A and goes in the direction of C.*

b *Name the segment joining A and B.*

Answers

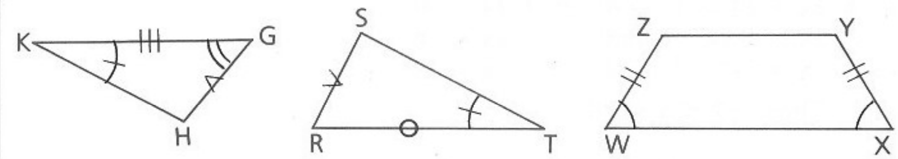
a $\overrightarrow{AB}$ or $\overrightarrow{AC}$

b $\overline{AB}$ or $\overline{BA}$

Problem 3 *Draw a diagram in which the intersection of* $\overrightarrow{AB}$ *with* $\overrightarrow{CA}$ *is* $\overline{AC}$ ($\overrightarrow{AB} \cap \overrightarrow{CA} = \overline{AC}$).

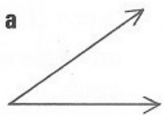
Solution

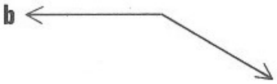
indicate that there are four pairs of congruent parts. Can you name them?

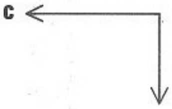


Part Two: Sample Problems

Problem 1 *Classify each of the angles below as acute, right, or obtuse. Then estimate the number of degrees in the angle.*

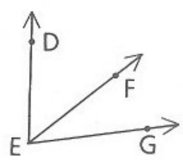
a


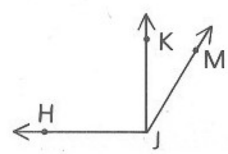
b


c


Answers **a** Acute; 40° **b** Obtuse; 150° **c** Right; 90°

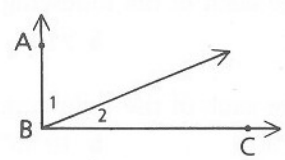
Problem 2 *In the diagram below,* ∠DEG = 80°, ∠DEF = 50°, ∠HJM = 120°, *and* ∠HJK = 90°. *Draw a conclusion about* ∠FEG *and* ∠KJM.





Solution ∠FEG = 30° and ∠KJM = 30°, so ∠FEG ≅ ∠KJM.

Problem 3 *Given:* ∠ABC *is a right angle.*
∠1 = (3x + 4)°,
∠2 = (x + 6)°
Find: m∠1 *(the measure of* ∠1*)*

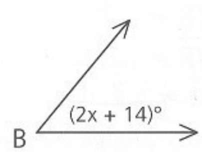


Solution Since ∠ABC is a right ∠, m∠1 + m∠2 = 90.

$$(3x + 4) + (x + 6) = 90$$
$$4x + 10 = 90$$
$$4x = 80$$
$$x = 20$$

Since m∠1 = 3x + 4, m∠1 = 3(20) + 4, or 64.

Problem 4 ∠B *is acute.*
a *What are the restrictions on* m∠B?
b *What are the restrictions on* x?



Solution

a Since $\angle B$ is acute, $m\angle B > 0$ and $m\angle B < 90$ ($0 < m\angle B < 90$).

b
$$\begin{aligned} 2x + 14 &> 0 &\text{and}&& 2x + 14 &< 90 \\ 2x &> -14 &\text{and}&& 2x &< 76 \\ x &> -7 &\text{and}&& x &< 38 \end{aligned}$$

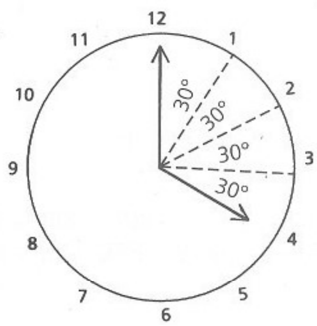
Thus, $-7 < x < 38$.

Problem 5 *Find the angle formed by the hands of a clock at each time.*

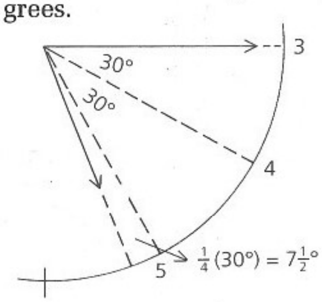
a 4:00 **b** 5:15

Solution

a Since 360° is divided into 12 intervals on a clock, each interval is 30°. From 12 to 4 there are 4 intervals, so the angle is 4(30°), or 120°.



b Remember that the hour hand is on 5 only when the minute hand is on 12. At 5:15 the hour hand is one fourth of the way from 5 to 6. Since $\frac{1}{4}(30°) = 7\frac{1}{2}°$, the hands form an angle of $60 + 7\frac{1}{2}$, or $67\frac{1}{2}$ degrees.



Part Three: Problem Sets

Problem Set A

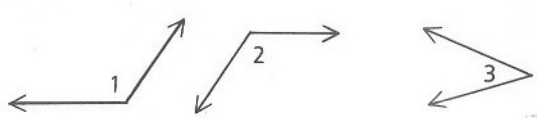
1 Change each of the following to degrees and minutes.

a $61\frac{2}{3}°$ **b** 71.7°

2 Change each of the following to degrees.

a 132°30′ **b** 19°45′

3 Which two of the angles below *appear* to be congruent?



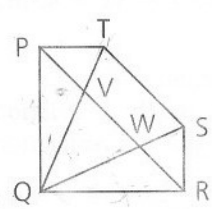
4 **a** $\overrightarrow{QV} \cap \overleftrightarrow{TS} = \underline{\ ?\ }$

b $\overline{WP} \cap \overline{VR} = \underline{\ ?\ }$

c $\overrightarrow{WP} \cup \overrightarrow{VR} = \underline{\ ?\ }$

d $\overrightarrow{SQ} \cup \overrightarrow{SR} = \underline{\ ?\ }$

e How many angles have vertex Q?



1.5 DIVISION OF SEGMENTS AND ANGLES

Objectives

After studying this section, you will be able to

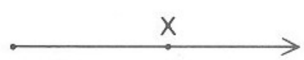
- Identify midpoints and bisectors of segments
- Identify trisection points and trisectors of segments
- Identify angle bisectors
- Identify angle trisectors

Part One: Introduction

Midpoints and Bisectors of Segments

We shall often work with segments that are divided in half.

Definition A point (or segment, ray, or line) that divides a segment into two congruent segments ***bisects*** the segment. The bisection point is called the ***midpoint*** of the segment.

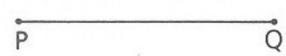


X is not a midpoint

Y is not a midpoint

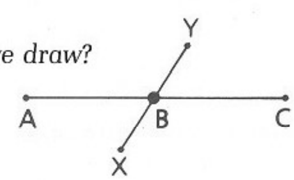
Only segments have midpoints. It does not make sense to say that a ray or a line has a midpoint. Do you understand why?

How many midpoints does $\overline{PQ}$ have?
How many bisectors could $\overline{PQ}$ have?



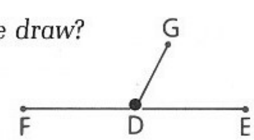
Study the following examples.

Example 1 *If* $\overline{XY}$ *bisects* $\overline{AC}$ *at* B, *what conclusions can we draw?*



Conclusions:
B is the midpoint of $\overline{AC}$.
$\overline{AB} \cong \overline{BC}$

Example 2 *If* D *is the midpoint of* $\overline{FE}$, *what conclusions can we draw?*



Conclusions:
$\overline{FD} \cong \overline{DE}$
Point D bisects $\overline{FE}$.
$\overline{DG}$ bisects $\overline{FE}$.

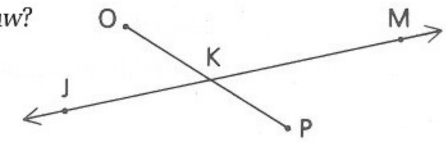
Example 3 *If* $\overline{OK} \cong \overline{KP}$, *what conclusions can we draw?*

Conclusions:

K is the midpoint of $\overline{OP}$.

$\overleftrightarrow{JM}$ is a bisector of $\overline{OP}$.

Point K bisects $\overline{OP}$.



Trisection Points and Trisecting a Segment

A segment divided into *three* congruent parts is said to be ***trisected***.

Definition Two points (or segments, rays, or lines) that divide a segment into three congruent segments ***trisect*** the segment. The two points at which the segment is divided are called the ***trisection points*** of the segment.

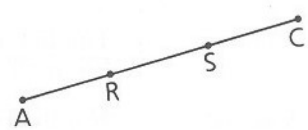
Again, only segments have trisection points; rays and lines do not have trisection points.

Example 1 *If* $\overline{AR} \cong \overline{RS} \cong \overline{SC}$, *what conclusions can we draw?*

Conclusions:

R and S are trisection points of $\overline{AC}$.

$\overline{AC}$ is trisected by R and S.

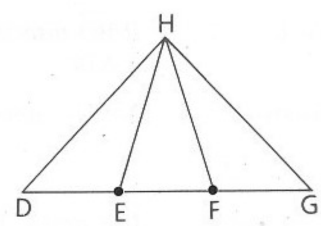


Example 2 *If* E *and* F *are trisection points of* $\overline{DG}$, *what conclusions can we draw?*

Conclusions:

$\overline{DE} \cong \overline{EF} \cong \overline{FG}$

$\overline{HE}$ and $\overline{HF}$ are trisectors of $\overline{DG}$.

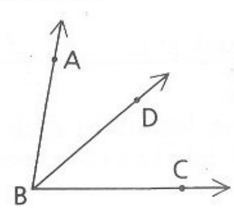


Angle Bisectors

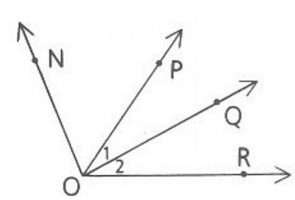
An angle, like a segment, can be bisected.

Definition A ray that divides an angle into two congruent angles ***bisects*** the angle. The dividing ray is called the ***bisector*** of the angle.

If $\angle ABD \cong \angle DBC$, then $\overrightarrow{BD}$ (not $\overrightarrow{DB}$) is the bisector of $\angle ABC$.



If $\angle NOP \cong \angle POR$ and $\overrightarrow{OQ}$ bisects $\angle POR$, then $\overrightarrow{OP}$ (not $\overrightarrow{PO}$) is the bisector of $\angle NOR$, and $\angle 1 \cong \angle 2$.

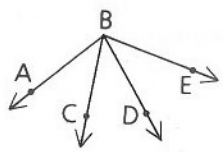


Angle Trisectors

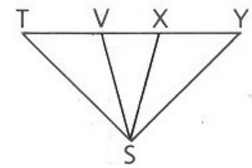
Two rays can divide an angle into three equal parts.

Definition Two rays that divide an angle into three congruent angles ***trisect*** the angle. The two dividing rays are called ***trisectors*** of the angle.

If $\angle ABC \cong \angle CBD \cong \angle DBE$, then $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.

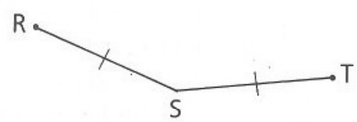


If $\overrightarrow{SV}$ and $\overrightarrow{SX}$ are trisectors of $\angle TSY$, then $\angle TSV \cong \angle VSX \cong \angle XSY$.



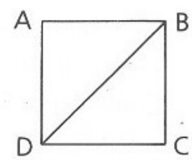
Part Two: Sample Problems

Problem 1 *The tick marks indicate that* $\overline{RS} \cong \overline{ST}$. *Is S the midpoint of* $\overline{RT}$?



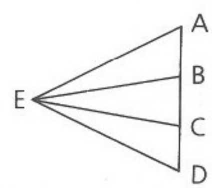
Answer No, the points are not collinear.

Problem 2 *If* $\overrightarrow{BD}$ *bisects* $\angle ABC$, *does* $\overrightarrow{DB}$ *bisect* $\angle ADC$?



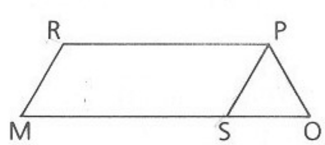
Answer No. We need more information.

Problem 3 *If* B *and* C *trisect* $\overline{AD}$, *do* $\overrightarrow{EB}$ *and* $\overrightarrow{EC}$ *trisect* $\angle AED$?



Answer No! It is true that $\overline{AB} \cong \overline{BC} \cong \overline{CD}$, but the fact that the *segment* has been trisected does not mean that the *angle* has been trisected.

Problem 4 *Given:* $\overrightarrow{PS}$ *bisects* $\angle RPO$.
Prove: $\angle RPS \cong \angle OPS$

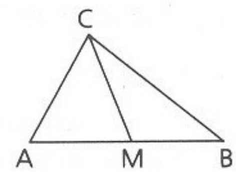


Proof

Statements	Reasons
1 $\overrightarrow{PS}$ bisects $\angle RPO$.	1 Given
2 $\angle RPS \cong \angle OPS$	2 If a ray bisects an angle, it divides the angle into two congruent angles.

Problem 5 *Given:* $\overleftrightarrow{CM}$ *bisects* $\overline{AB}$ *(In Chapter 3 we shall call* $\overline{CM}$ *a* median *of the triangle.)*

Conclusion: $\overline{AM} \cong \overline{MB}$

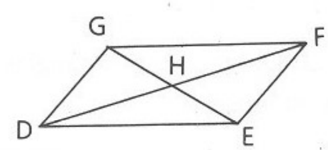


Proof

Statements	Reasons
1 $\overleftrightarrow{CM}$ bisects $\overline{AB}$.	1 Given
2 $\overline{AM} \cong \overline{MB}$	2 If a line bisects a segment, it divides the segment into two congruent segments.

Problem 6 *Given:* $\overline{DH} \cong \overline{HF}$

Prove: H *is the midpoint of* $\overline{DF}$.



Proof

Statements	Reasons
1 $\overline{DH} \cong \overline{HF}$	1 Given
2 H is the midpoint of $\overline{DF}$.	2 If a point divides a segment into two congruent segments, it is the midpoint of the segment.

Problem 7 $\overline{EH}$ *is divided by* F *and* G *in the ratio* 5:3:2 *from left to right. If* EH = 30, *find* FG *and name the midpoint of* $\overline{EH}$.

Solution According to the ratio, we can let EF = 5x, FG = 3x, and GH = 2x. First we draw a diagram and place the algebra on it as part of the solution.

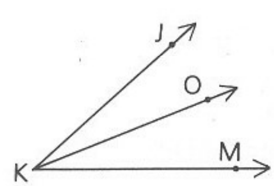
$$5x + 3x + 2x = 30$$
$$10x = 30$$
$$x = 3$$

Thus, FG = 3(3), or 9. Since EF = 15 and FH = 15, F is the midpoint of $\overline{EH}$.

Problem 8 *Given:* $\overrightarrow{KO}$ *bisects* $\angle$JKM.

$\angle JKM = 41°37'$

Find: m$\angle$OKM



Solution $\frac{1}{2}(41°37') = 20\frac{1}{2}°18\frac{1}{2}'$

$= 20°48\frac{1}{2}'$ (since $\frac{1}{2}° = 30'$)

$= 20°48'30''$ (since $\frac{1}{2}' = 30''$)