12-2 Chords and Arcs

TEKS FOCUS

TEKS (12)(A) Apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(A), (1)(G), (5)(A), (5)(C), (6)(A), (9)(B)

VOCABULARY

- **Chord** – a segment whose endpoints are on a circle
- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

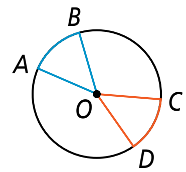
You can use information about congruent parts of a circle (or congruent circles) to find information about other parts of the circle (or circles).

take note

Theorem 12-4 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent central angles.



If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

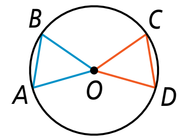
If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong \angle COD$.

You will prove Theorem 12-4 and its converse in Exercises 7 and 24.

Theorem 12-5 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent chords.

Converse
Within a circle or in congruent circles, congruent chords have congruent central angles.



If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

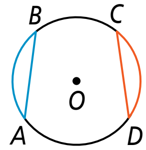
You will prove Theorem 12-5 and its converse in Exercises 8 and 25.

take note

Theorem 12-6 and Its Converse

Theorem
Within a circle or in congruent circles, congruent chords have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent chords.



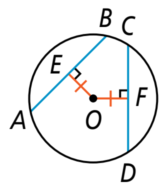
If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.
If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.

You will prove Theorem 12-6 and its converse in Exercises 9 and 26.

Theorem 12-7 and Its Converse

Theorem
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

Converse
Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).



If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.
If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.

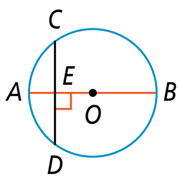
For a proof of Theorem 12-7, see the Reference section on page 683.
You will prove the converse of Theorem 12-7 in Exercise 27.

take note

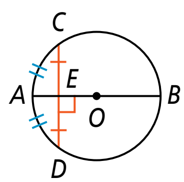
Theorem 12-8

Theorem
In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

If . . .
$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$



Then . . .
$\overline{CE} \cong \overline{ED}$ and $\overset{\frown}{CA} \cong \overset{\frown}{AD}$

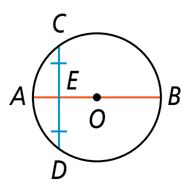


You will prove Theorem 12-8 in Exercise 10.

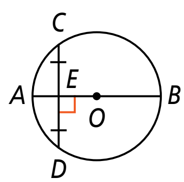
Theorem 12-9

Theorem
In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

If . . .
$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$



Then . . .
$\overline{AB} \perp \overline{CD}$



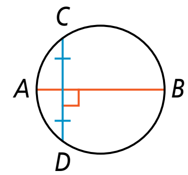
For a proof of Theorem 12-9, see the Reference section on page 683.

Theorem 12-10

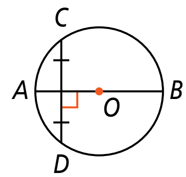
take note

Theorem
In a circle, the perpendicular bisector of a chord contains the center of the circle.

If . . .
$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$



Then . . .
$\overline{AB}$ contains the center of $\odot O$

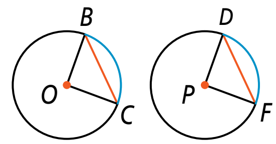


You will prove Theorem 12-10 in Exercise 11.

Problem 1

Using Congruent Chords

In the diagram, $\odot O \cong \odot P$. Given that $\overline{BC} \cong \overline{DF}$, what can you conclude?



$\angle O \cong \angle P$ because, within congruent circles, congruent chords have congruent central angles (conv. of Thm. 12-5). $\overset{\frown}{BC} \cong \overset{\frown}{DF}$ because, within congruent circles, congruent chords have congruent arcs (Thm. 12-6).

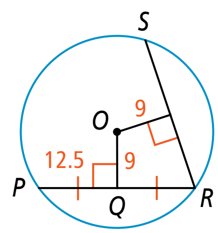
Think

Why is it important that the circles are congruent?
The central angles will not be congruent unless the circles are congruent.

Problem 2

Finding the Length of a Chord

What is the length of $\overline{RS}$ in $\odot O$?

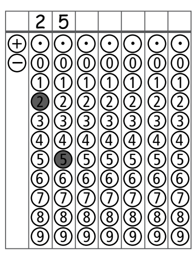


Know
The diagram indicates that $PQ = QR = 12.5$ and $\overline{PR}$ and $\overline{RS}$ are both 9 units from the center.

Need
The length of chord $\overline{RS}$

Plan
$\overline{PR} \cong \overline{RS}$, since they are the same distance from the center of the circle. So finding PR gives the length of $\overline{RS}$.

$PQ = QR = 12.5$	Given in the diagram
$PQ + QR = PR$	Segment Addition Postulate
$12.5 + 12.5 = PR$	Substitute.
$25 = PR$	Add.
$RS = PR$	Chords equidistant from the center of a circle are congruent (Theorem 12-7).
$RS = 25$	Substitute.



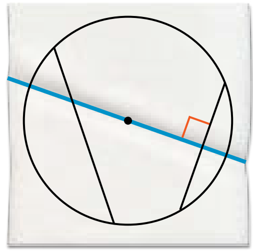
Problem 3

TEKS Process Standard (1)(C)

Investigating Special Segments of Circles

Choose from a variety of tools (such as a compass, straightedge, geometry software, and pencil and paper) to investigate the perpendicular bisectors of chords of a circle. Draw a circle with two chords that are not diameters. Construct the perpendicular bisectors of the chords. Then make a conjecture about the perpendicular bisector of a chord.

You can construct perpendicular bisectors using paper, a pencil, a compass, and a straightedge. You can draw the circle and chords using a compass and straightedge, and construct the perpendicular bisectors using paper folding.



Step 1 Use a compass to draw a circle on a piece of paper.

Step 2 Use a straightedge to draw two chords that are not diameters.

Step 3 Fold the perpendicular bisector for each chord. The perpendicular bisectors appear to intersect at the center of the circle.

Step 4 Draw a third chord and construct its perpendicular bisector. The third perpendicular bisector also appears to intersect the other two.

Conjecture: The perpendicular bisector of any chord of a circle goes through the center of the circle.

Think

Why should you draw more than two chords?
The more examples you can find to support your conjecture, the stronger your conjecture becomes.

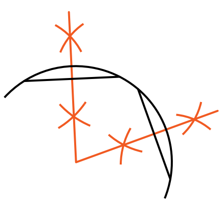
Problem 4

TEKS Process Standard (1)(A)

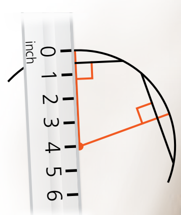
Using Diameters and Chords

Archaeology **An archaeologist found pieces of a jar. She wants to find the radius of the rim of the jar to help guide her as she reassembles the pieces. What is the radius of the rim?**

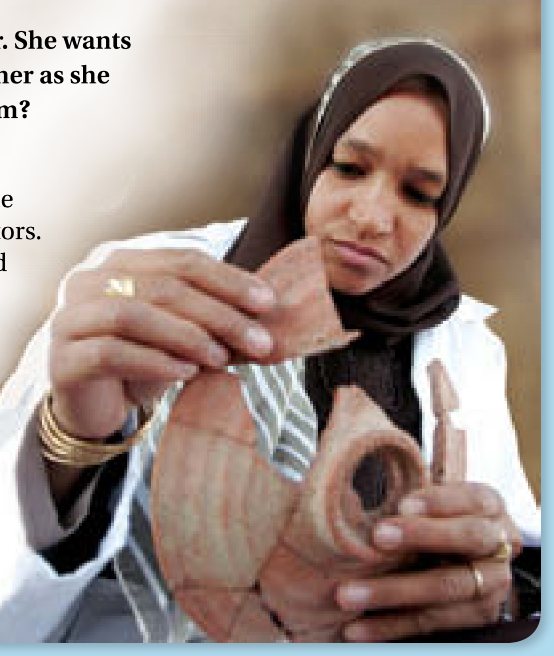
Step 1 Trace a piece of the rim. Draw two chords and construct perpendicular bisectors.



Step 2 The center is the intersection of the perpendicular bisectors. Use the center to find the radius.



The radius is 4 in.



Think

How does the construction help find the center?
The perpendicular bisectors contain diameters of the circle. Two diameters intersect at the circle's center.

Problem 5

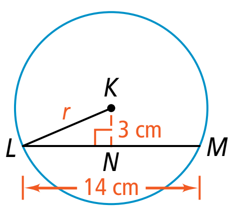
Finding Measures in a Circle

Plan

Find two sides of a right triangle. The third side either is the answer or leads to an answer.

Algebra **What is the value of each variable to the nearest tenth?**

A

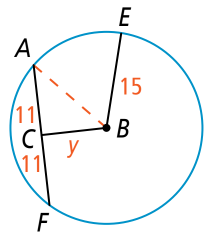


$LN = \frac{1}{2}(14) = 7$ A diameter $\perp$ to a chord bisects the chord (Theorem 12-8).

$r^2 = 3^2 + 7^2$ Use the Pythagorean Theorem.

$r \approx 7.6$ Find the positive square root of each side.

B



$\overline{BC} \perp \overline{AF}$ A diameter that bisects a chord that is not a diameter is $\perp$ to the chord (Theorem 12-9).

$BA = BE = 15$ Draw an auxiliary $\overline{BA}$. The auxiliary $\overline{BA} \cong \overline{BE}$ because they are radii of the same circle.

$y^2 + 11^2 = 15^2$ Use the Pythagorean Theorem.

$y^2 = 104$ Solve for y^2.

$y \approx 10.2$ Find the positive square root of each side.

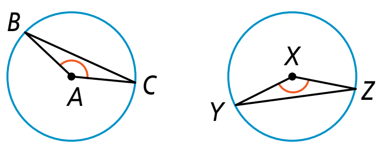
PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

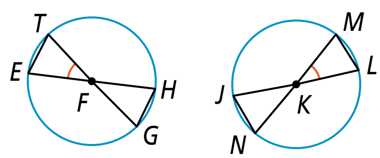
For additional support when completing your homework, go to PearsonTEXAS.com.

In Exercises 1 and 2, the circles are congruent. What can you conclude?

1.

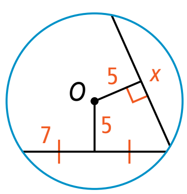


2.

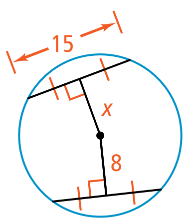


Find the value of *x*.

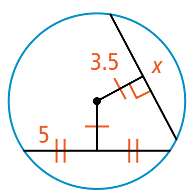
3.



4.



5.



6. **Justify Mathematical Arguments (1)(G)** In the diagram at the right, $\overline{GH}$ and $\overline{KM}$ are perpendicular bisectors of the chords they intersect. What can you conclude about the center of the circle? Justify your answer.