12-1 Tangent Lines

TEKS FOCUS

TEKS (12)(A) Apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems.

TEKS (1)(F) **Analyze** mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(G), (6)(A), (9)(B)

VOCABULARY

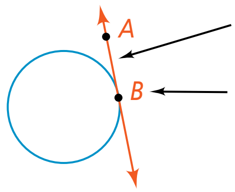
- **Point of tangency** – the point where a circle and a tangent intersect
- **Tangent to a circle** – a line in the plane of the circle that intersects the circle in exactly one point

- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature

ESSENTIAL UNDERSTANDING

A radius of a circle and the tangent that intersects the endpoint of the radius on the circle have a special relationship.

take note — Key Concept Tangent Lines



A **tangent to a circle** is a line in the plane of the circle that intersects the circle in exactly one point.

The point where a circle and a tangent intersect is the **point of tangency.**

$\overrightarrow{BA}$ is a tangent ray, and $\overline{BA}$ is a tangent segment.

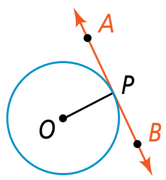
take note — Theorem 12-1

Theorem

If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.

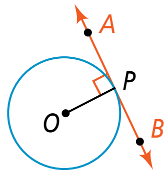
If . . .

$\overleftrightarrow{AB}$ is tangent to $\odot O$ at P



Then . . .

$\overleftrightarrow{AB} \perp \overline{OP}$

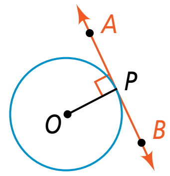


For a proof of Theorem 12-1, see the Reference section on page 683.

Theorem 12-2

Theorem
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

If . . .
$\overleftrightarrow{AB} \perp \overline{OP}$ at P



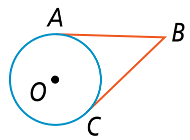
Then . . .
$\overleftrightarrow{AB}$ is tangent to $\odot O$

You will prove Theorem 12-2 in Exercise 19.

Theorem 12-3

Theorem
If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

If . . .
$\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$



Then . . .
$\overline{BA} \cong \overline{BC}$

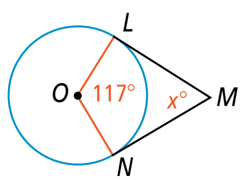
You will prove Theorem 12-3 in Exercise 12.

Problem 1

TEKS Process Standard (1)(F)

Finding Angle Measures

Multiple Choice $\overline{ML}$ **and** $\overline{MN}$ **are tangent to** $\odot O$**. What is the value of** x**?**



Ⓐ 58 Ⓑ 63 Ⓒ 90 Ⓓ 117

Think

What kind of angle is formed by a radius and a tangent?
The angle formed is a right angle, so the measure is 90.

Since $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot O$, $\angle L$ and $\angle N$ are right angles (Theorem 12-1). $LMNO$ is a quadrilateral. So the sum of the angle measures is 360.

$$m\angle L + m\angle M + m\angle N + m\angle O = 360$$

$$90 + m\angle M + 90 + 117 = 360 \quad \text{Substitute.}$$

$$297 + m\angle M = 360 \quad \text{Simplify.}$$

$$m\angle M = 63 \quad \text{Solve.}$$

The correct answer is B.

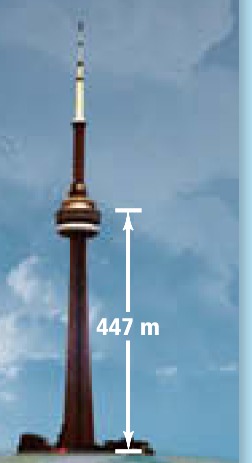
Problem 2

Finding Distance STEM

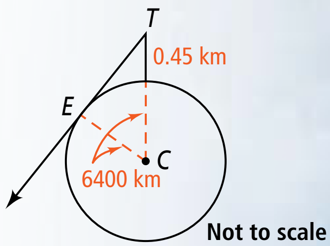
Plan

How does knowing Earth's radius help?
The radius forms a right angle with a tangent line from the observation deck to the horizon. So you can use two radii, the tower's height, and the tangent to form a right triangle.

Earth Science The CN Tower in Toronto, Canada, has an observation deck 447 m above ground level. About how far is it from the observation deck to the horizon? Earth's radius is about 6400 km.



Step 1 Make a sketch. The length 447 m is about 0.45 km.



Step 2 Use the Pythagorean Theorem.

$CT^2 = TE^2 + CE^2$	
$(6400 + 0.45)^2 = TE^2 + 6400^2$	Substitute.
$(6400.45)^2 = TE^2 + 6400^2$	Simplify.
$40{,}965{,}760.2025 = TE^2 + 40{,}960{,}000$	Use a calculator.
$5760.2025 = TE^2$	Subtract 40,960,000 from each side.
$76 \approx TE$	Take the positive square root of each side.

The distance from the CN Tower to the horizon is about 76 km.

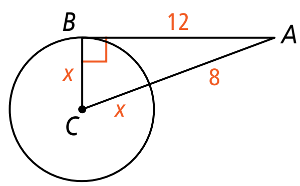
Problem 3

Finding a Radius

Think

Why does the value x appear on each side of the equation?
The length of $\overline{AC}$, the hypotenuse, is the radius plus 8, which is on the left side of the equation. On the right side of the equation, the radius is one side of the triangle.

What is the radius of ⊙C?



$AC^2 = AB^2 + BC^2$	Pythagorean Theorem
$(x + 8)^2 = 12^2 + x^2$	Substitute.
$x^2 + 16x + 64 = 144 + x^2$	Simplify.
$16x = 80$	Subtract x^2 and 64 from each side.
$x = 5$	Divide each side by 16.

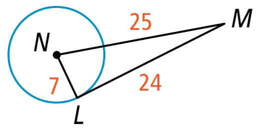
The radius is 5.

Problem 4

TEKS Process Standard (1)(G)

Identifying a Tangent

Is $\overline{ML}$ tangent to $\odot N$ at L? Explain.



Think

What information does the diagram give you?
- LMN is a triangle.
- $NM = 25$, $LM = 24$, $NL = 7$
- $\overline{NL}$ is a radius.

Know

The lengths of the sides of $\triangle LMN$

Need

To determine whether $\overline{ML}$ is tangent to $\odot N$

Plan

$\overline{ML}$ is a tangent if $\overline{ML} \perp \overline{NL}$. Use the Converse of the Pythagorean Theorem to determine whether $\triangle LMN$ is a right triangle.

$NL^2 + ML^2 \stackrel{?}{=} NM^2$

$7^2 + 24^2 \stackrel{?}{=} 25^2$ Substitute.

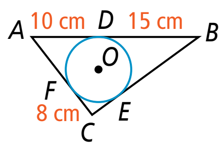
$625 = 625$ Simplify.

By the Converse of the Pythagorean Theorem, $\triangle LMN$ is a right triangle with $\overline{ML} \perp \overline{NL}$. So $\overline{ML}$ is tangent to $\odot N$ at L because it is perpendicular to the radius at the point of tangency (Theorem 12-2).

Problem 5

Circles Inscribed in Polygons

$\odot O$ is inscribed in $\triangle ABC$. What is the perimeter of $\triangle ABC$?



Plan

How can you find the length of $\overline{BC}$?
Find the segments congruent to $\overline{BE}$ and $\overline{EC}$. Then use segment addition.

$AD = AF = 10$ cm
$BD = BE = 15$ cm
$CF = CE = 8$ cm

Thm 12-3: Two segments tangent to a circle from a point outside the circle are congruent, so they have the same length.

$P = AB + BC + CA$ Definition of perimeter

$= AD + DB + BE + EC + CF + FA$ Segment Addition Postulate

$= 10 + 15 + 15 + 8 + 8 + 10$ Substitute.

$= 66$

The perimeter is 66 cm.