12-3 Inscribed Angles

VOCABULARY

- **Inscribed angle** – an angle whose vertex is on the circle and whose sides are chords of the circle

- **Justify** – explain with logical reasoning. You can justify a mathematical argument.

- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim
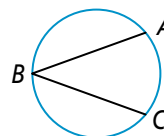
ESSENTIAL UNDERSTANDING

Angles formed by intersecting lines have a special relationship to the arcs the intersecting lines intercept. In this lesson, you will study arcs formed by inscribed angles.

take note

Theorem 12-11 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.
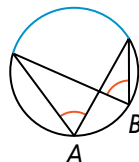
$$m\angle B = \frac{1}{2}\,m\widehat{AC}$$

You will prove Theorem 12-11 in Exericises 9 and 10.

take note

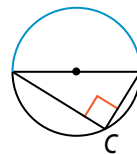## Corollaries to Theorem 12-11: The Inscribed Angle Theorem

Corollary 1
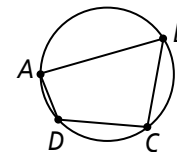Two inscribed angles that intercept the same arc are congruent.

Corollary 2
An angle inscribed in a semicircle is a right angle.

Corollary 3
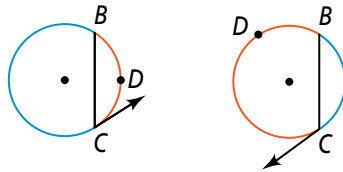The opposite angles of a quadrilateral inscribed in a circle are supplementary.

You will prove these corollaries in Exercises 14–16.

take note

Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

$$m\angle C = \tfrac{1}{2}\, m\widehat{BDC}$$

You will prove Theorem 12-12 in Exercise 17.

Problem 1

Using the Inscribed Angle Theorem

Plan

Which variable should you solve for first?
You know the inscribed angle that intercepts $\widehat{PT}$, which has the measure a. You need a to find b. So find a first.

What are the values of *a* and *b*?

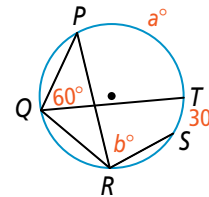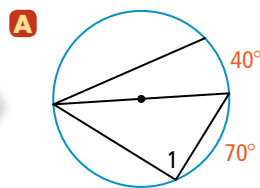|	
$m\angle PQT = \tfrac{1}{2}\, m\widehat{PT}$	Inscribed Angle Theorem
$60 = \tfrac{1}{2}\, a$	Substitute.
$120 = a$	Multiply each side by 2.
$m\angle PRS = \tfrac{1}{2}\, m\widehat{PS}$	Inscribed Angle Theorem
$m\angle PRS = \tfrac{1}{2}\,(m\widehat{PT} + m\widehat{TS})$	Arc Addition Postulate
$b = \tfrac{1}{2}\,(120 + 30)$	Substitute.
$b = 75$	Simplify.

Problem 2

Using Corollaries to Find Angle Measures

Think

Is there too much information?
Each diagram has more information than you need. Focus on what you need to find.

What is the measure of each numbered angle?

A

B

$\angle 1$ is inscribed in a semicircle. By Corollary 2, $\angle 1$ is a right angle, so $m\angle 1 = 90$.
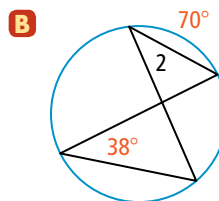
$\angle 2$ and the $38°$ angle intercept the same arc. By Corollary 1, the angles are congruent, so $m\angle 2 = 38$.