12-4 Angle Measures and Segment Lengths

TEKS FOCUS

TEKS (12)(A) Apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(D), (1)(F), (5)(A), (6)(A), (6)(D)

VOCABULARY

- **Secant** – a line that intersects a circle at two points
- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

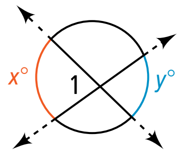
- Angles formed by intersecting lines have a special relationship to the related arcs formed when the lines intersect a circle.
- There is a special relationship between two intersecting chords, two intersecting secants, or a secant that intersects a tangent. This relationship allows you to find the lengths of unknown segments.

take note

Theorem 12-13

The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

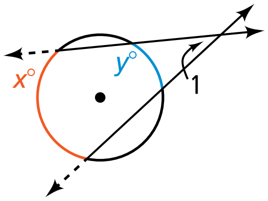
$$m\angle 1 = \frac{1}{2}(x + y)$$



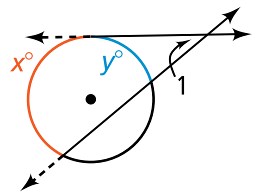
For a proof of Theorem 12-13, see the Reference section on page 683.

Theorem 12-14

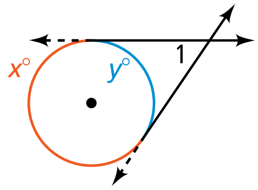
The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.



I: angle formed by two secants



II: angle formed by a secant and a tangent



III: angle formed by two tangents

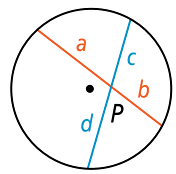
$$m\angle 1 = \frac{1}{2}(x - y)$$

You will prove Theorem 12-14 in Exercises 24 and 25.

take note Theorem 12-15

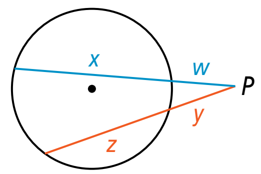
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.

I.



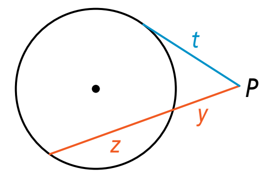
$a \cdot b = c \cdot d$

II.



$(w + x)w = (y + z)y$

III.



$(y + z)y = t^2$

You will prove Theorem 12-13 in Exercises 13 and 14.

Problem 1

TEKS Process Standard (1)(C)

Investigating Special Angles of Circles

Choose from a variety of tools (such as a protractor, a compass, or geometry software) to investigate angles formed by chords intersecting inside a circle. Explain why you chose that tool. Construct a circle and several pairs of intersecting chords. Then make a conjecture about the measure of an angle formed by two chords intersecting inside a circle.

Geometry software could help you investigate angles created by chords. You can create a circle and two chords and then drag the chords around the circle to investigate angle measures and make a conjecture.

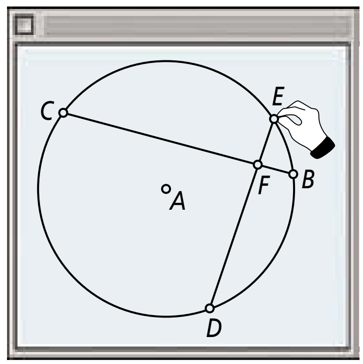
Think

Which angles are formed by the intersection of the chords in the diagram?
$\angle CFD$, $\angle EFB$, $\angle CFE$, and $\angle DFB$

Step 1 Draw a circle and two chords that intersect in the interior of the circle.

Step 2 Measure angles formed by the intersection of the chords and their intercepted arcs. Record your findings in the table below.

Step 3 Drag the chords to form different angles. Record your findings in a table.



Vertical angle measure	65°	41°	81°
Intercepted arc	44°	59°	30°
Intercepted arc	86°	23°	132°
Sum of arc measures	130°	82°	162°

Conjecture: The measure of an angle formed by two chords that intersect inside a circle is half the sum of the measures of its intercepted arcs.

Problem 2

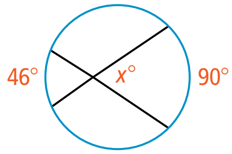
Finding Angle Measures

Algebra **What is the value of each variable?**

Think

Remember to add arc measures for arcs intercepted by lines that intersect inside a circle and subtract arc measures for arcs intercepted by lines that intersect outside a circle.

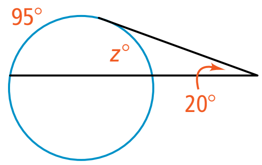
A



$x = \frac{1}{2}(46 + 90)$ Theorem 12-13

$x = 68$ Simplify.

B



$20 = \frac{1}{2}(95 - z)$ Theorem 12-14

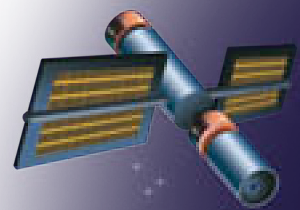
$40 = 95 - z$ Multiply each side by 2.

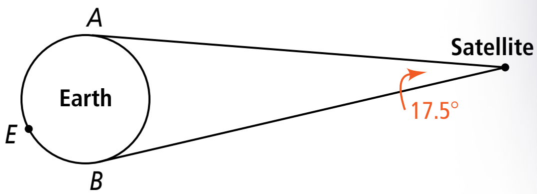
$z = 55$ Solve for z.

Problem 3

Finding an Arc Measure

Satellite **A satellite in a geostationary orbit above Earth's equator has a viewing angle of Earth formed by the two tangents to the equator. The viewing angle is about 17.5°. What is the measure of the arc of Earth that is viewed from the satellite?**





Think

How can you represent the measures of the arcs?
The sum of the measures of the arcs is 360°. If the measure of one arc is x, then the measure of the other is $360 - x$.

Let $m\widehat{AB} = x$.

Then $m\widehat{AEB} = 360 - x$.

$17.5 = \frac{1}{2}(m\widehat{AEB} - m\widehat{AB})$ Theorem 12-14

$17.5 = \frac{1}{2}[(360 - x) - x]$ Substitute.

$17.5 = \frac{1}{2}(360 - 2x)$ Simplify.

$17.5 = 180 - x$ Distributive Property

$x = 162.5$ Solve for x.

A 162.5° arc can be viewed from the satellite.

Problem 4

TEKS Process Standard (1)(F)

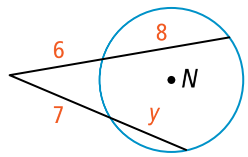
Finding Segment Lengths

Plan

How can you identify the segments needed to use Theorem 12-15? Find where segments intersect each other relative to the circle. The lengths of segments that are part of one line will be on the same side of an equation.

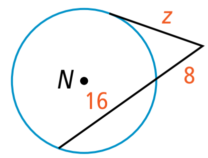
Algebra **Find the value of the variable in ⊙*N*.**

A



$(6 + 8)6 = (7 + y)7$	Thm. 12-15, Case II
$84 = 49 + 7y$	Distributive Property
$35 = 7y$	
$5 = y$	Solve for *y*.

B



$(8 + 16)8 = z^2$	Thm. 12-15, Case III
$192 = z^2$	Simplify.
$13.9 \approx z$	Solve for *z*.

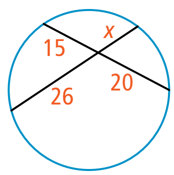
PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to **PearsonTEXAS.com**.

Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.

1.



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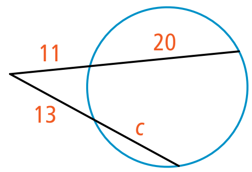
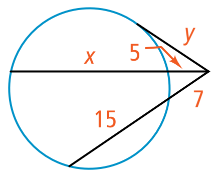
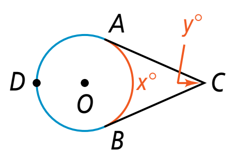


3.



In the diagram at the right, $\overline{CA}$ and $\overline{CB}$ are tangents to ⊙*O*. Write an expression for each arc or angle in terms of the given variable.



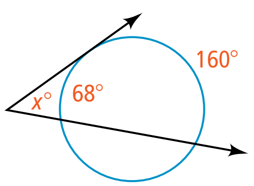
4. $m\widehat{ADB}$ using x

5. $m\angle C$ using x

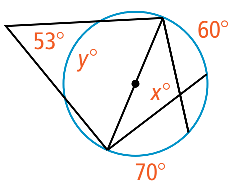
6. $m\widehat{AB}$ using y

Find the value of each variable.

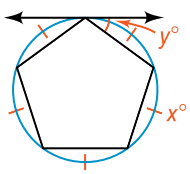
7.



8.



9.



10. Analyze Mathematical Relationships (1)(F) You focus your camera on a circular fountain. Your camera is at the vertex of the angle formed by tangents to the fountain. You estimate that this angle is 40°. What is the measure of the arc of the circular basin of the fountain that will be in the photograph?